



**JUUSO TÖYLI** • SENIOR RESEARCHER • LABORATORY OF COMPUTATIONAL ENGINEERING  
• HELSINKI UNIVERSITY OF TECHNOLOGY

**KIMMO KASKI** • ACADEMY PROFESSOR  
• LABORATORY OF COMPUTATIONAL ENGINEERING • HELSINKI UNIVERSITY OF TECHNOLOGY

**ANTTI KANTO** • PROFESSOR • DEPARTMENT OF ECONOMICS AND MANAGEMENT SCIENCE  
• HELSINKI SCHOOL OF ECONOMICS

## Variability of Hex Return Distribution

This paper discussed the shape of the stock return distribution using the all shares index of Helsinki Stock Exchange. Non-parametric kernel density estimation and power exponential family of distributions were used to model the shape of the return distribution. The parameters of power exponential distribution were estimated with Bayesian approach. In general, the findings suggest that the shape of the distribution does not vary from one weekday to another. However, substantial deviations over time are observed while there are also temporal periods when it is reasonable to assume the return generating process as normal. Furthermore, the results indicate that the return distribution approaches normal when the time interval used to calculate returns is increased.


The results derived with kernel density estimation indicated that return distribution is unimodal, leptokurtic, and quite symmetric. However, the limited amount of data available

made the tails of the kernel density estimate messy and, thus, no inference concerning these was possible. In addition, it seems, given the sample size, that the kernel density estimator underestimates significantly the probability mass of the central regions. These difficulties lead to the conclusion that kernel density estimation might not yield a good description of the distribution – at least the statistical sample size should be substantially larger.

The second model, we used, was the power exponential family of distributions. Their parameters were estimated with the Bayesian approach. The location parameter  $\theta$  seemed to be basically zero although, in statistical sense, small but significant deviations were observed. These deviations have little meaning in reality since, given the transaction costs, it is hardly possible to place any profitable trading rules on them. It also seems that, in case of short-term inference, it would be justifiable to assume the location parameter to be zero. On the other hand, the long-run expectation was slightly positive as was expected. No significant deviations between weekdays were observed although the expectation tends to somewhat increase towards the end of the week being negative on Mondays.

The major issue was the shape of the distribution. Since the return series generally indicate anomalies related to weekdays, the question whether the shape of the return distribution differs between weekdays was explored, but no convincing evidence about the differences in the shape of the distribution between weekdays was found. This result might not hold in other markets because the weekday effects are not generally strong on Finnish stock market.

The variability of the shape of the distribution over time was especially interesting



## EXECUTIVE SUMMARIES

since the analyses provided strong evidence that the shape is not constant. Along with that, periods, when the return generating process is normal, were observed. This suggests that part of the contradiction in the previous results concerning the shape of the distribution might be explained by the variability of the shape of the distribution over time. The shape of return distribution also seemed to approach steadily normal distribution when the time interval used to calculate returns was increased from one day towards thirty days' returns. This observation is also consistent result with the previous research. ■