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An Explanation of the Unusual Behavior of Some Market Model Residuals

ABSTRACT

Stock price behavior is often described via the market model, in which the residual term is presumed to measure abnormal returns available to the investor. This error term is assumed to: (1) have a mean of zero and (2) be normally distributed, with a constant variance. Empirical tests, however, sometimes report abnormal returns, whose distributions violate the assumptions which underlie them. The research reported here argues that such deviations might be explained by the presence of a bimodal error term. The existence of bimodality would help explain those observations in which: (1) cumulative abnormal returns are not zero and (2) levels of confidence are lower than expected. In addition, the bimodal model helps explain the variability of post-event adjustment lag.

Keywords: *Bimodality; Cumulative Abnormal Returns; Interim Reports; Market Model*

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INTRODUCTION

In a semi-strong efficient market, the assessment of unexpected disclosures is commonly assumed to yield cumulative abnormal returns (CARs) that are normally distributed with a constant variance about a mean of zero. Empirically, the explanatory power of this theory is often rather low: (1) the mean CAR is not always zero and (2) its associated distribution is not always homoscedastic or even bell-shaped. This article postulates one reason for this *apparent* discrepancy: the wrong assumptions are sometimes made about the distribution of the CARs. In certain cases, CARs may be bimodal. This would explain the thick tails that appear when the unimodal distribution such as a t-distribution is used to describe it. Further, unless the two modes have exactly the same probability density, a single mode model would generate a CAR whose mean is not zero.

CAR distributions with two modes, if they exist, are transitory. As further information is received, a market consensus developes around one or the other of the two causes of the bimodal CARs. This eliminates one of the modes. Market consensus results in a stable condition, that can then be appropriately described by a standard, single mode model. An attendant result is an adjustment lag of varying duration, as the market grasp for an understanding of which mode (favorable or unfavorable) will better describe expected future events. This article first develops a theoretical model to describe the phenomenon. This model helps to explain: (1) CARs that are not centered at zero, (2) wide CAR distributions and (3) variability in adjustment lags (post-event anomaly). Next, empirical tests confirm the reasonabless of the argument, *via* the observation of stock price adjustments to news reported in interim reports.

BACKGROUND

As early as 1982, Jain recognizes the problem by reporting that heterogeneous variances of market model residuals introduce bias in the estimates produced by that model. His answer is to develop a model that explicitly reduces the effect of the dependence among abnormal returns across firms. He uses the unimodal residual model, both to identify and mitigate the problem.

Neuburger & Stokes (1991) echo concern about the assumptions underlying the distribution of abnormal returns. They report that results which are based on an error term which is assumed to have a mean of zero and a normal distribution can be misleading.

A year earlier, Vaga (1990) suggests the idea which produces the solution that is presented in this study. His work is a purely theoretical treatise on stock market movements. He demonstrates that market movements can be described mathematically by changes in: (1) investor (crowd) sentiment and (2) the underlying economic conditions. Four stages of his very fluid model are given special attention.

His first example is equivalent to the mean of zero, normal distribution case. This result is obtained with the added assumptions of: (1) relatively little investor expectation of major changes (neutral sentiment) and (2) a neutral economy. The market returns center about zero and are bell-shaped. This is an expression of a ”true random walk,” and it can be fairly stable.

He calls his second case ”unstable transition.” This occurs when: (1) investor sentiment is restive with (2) a neutral economy. This situation is not stable. The integrals of his theoretical model yield a distribution that is, temporarily, almost horizontal.

Vaga’s third illustration is called the ”coherent market.” This condition occurs when: (1) investor behavior is highly bullish and (2) the economy is strong. The resulting unimodal distribution is centered to the right of zero and is skewed to the left. This favorable result is a stable condition in which returns are greater than zero and the distribution is relatively tight. This leads to a prolonged condition in which returns are high and risks are low.

The fourth example is the one that is mirrored in this research. Vaga calls it a ”chaotic market.” It exists when: (1) investor behavior is strongly negative and (2) the underlying economy is weak. This is an unstable condition, because it results in a bimodal distribution. The center of the distribution is not necessarily zero. The larger lobe is located to the left of the zero. This results, for a short time, in negative returns and very great risk. This is a description of a bear market. Because it is unstable, this condition will ultimately resolve itself into a unimodal distribution, centered at one or the other of the two maxima. An example of the shape of a general normal distribution and its normal approximation with bimodality is presented in figure I.

For emphasis, figure I shows an exagerated case. The taller peak is the result of the activity of investors who expect a favorable future. The smaller peak is created by the activity of investors who expect an unfavorable future. This illustration makes it clear that the normal approximation seriously biases measures of location, dispursion, skew and kurtosis. When standard regression techniques are applied, a bimodal error distribution appears as a leptokurtic distribution that is not usually centered on zero.

Vaga reports his theoretical model of the dynamic behavior of markets in 1990. That same

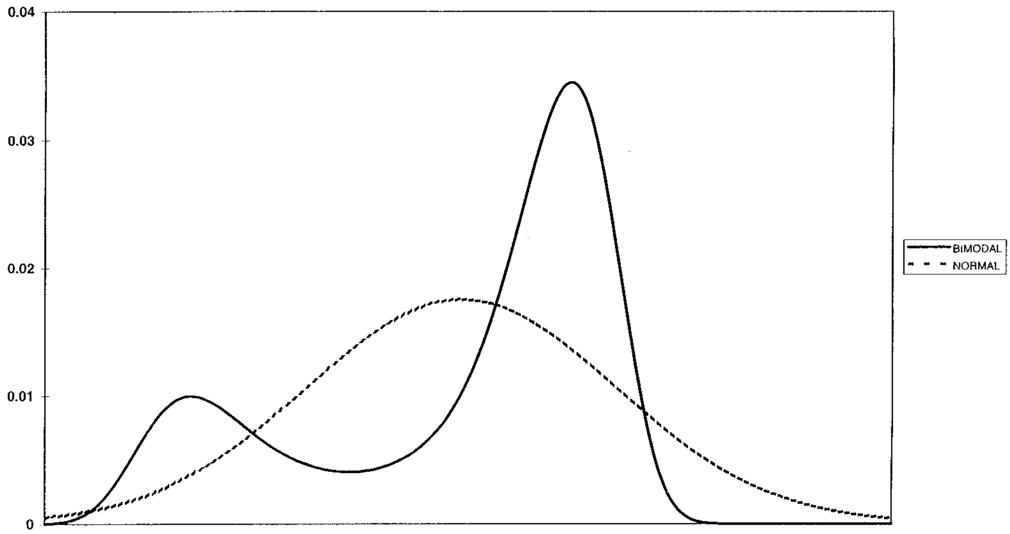

FIGURE 1. Typical shape of a bimodal density

year, Martin (1990) provides a complete discussion of the mathematical properties of the exponential functions associated with processes similar to those described by Vaga. Like the Vaga article, the Martin dissertation is a theoretical piece. Thus, in the first year of the decade of the 1990s, the tools are in place to answer two questions raised by the behavior of the market model. One, why is the mean CAR sometimes not zero? Two, why are the levels of confidence associated with CAR sometimes so low? The answer to a third question is implied by the work done by Vaga and Martin. That question is related to the puzzling appearance of post-announcement drift (Ball & Bartov 1996). The question is: what causes the *variations* in the adjustment lag which sometimes appear?

The Vaga (1990) theory and the Martin (1990) methodology are successfully applied by Kahra (1997) to model the FIM/DEM exchange rate.

MODEL

Jain (1982) and Neuburger & Stokes (1991) raise two market model based issues: (1) the distribution of some residual terms is not centered at zero and (2) the level of confidence associated with the residuals is sometimes very low. The application reported in this research uses the Vaga (1990) theory and the Martin (1990) methodology to help answer these questions. In addition, the model attacks the problem of variability in adjustment lags in order to partially explain the post-announcement-drift phenomenon reported by Ball & Bartov (1996).

Vaga's "chaotic market" presents a bimodal distribution that exists for a short time. This research argues that new investor information sometimes creates a very similar result. The empirical cases reported in this research are those which arise when interim reports contain value determining information that is both good and bad, and at least some of it is news to the market. This format allows the expression of the Vaga/Martin theoretical model in a well known context that is subject to testing.

Equation (1) presents the Fama (1976) expression of the market model:

(1) $R_{it} = \alpha_i + \beta_i R_{mt} + AR_{it}$

The market return (R_{mt}) is assumed to specify the individual security return (R_{it}) with the residual representing an abnormal return (AR_{it}). The expected value of AR_{it} is zero, bounded by a homoscedastic, bell-shaped distribution.

It follows that the cumulation of these AR_{it} values (CARs) will tend to be zero. Any departure from that would represent an inefficiency in the timing of adjustment or an ineffectiveness in the magnitude or direction of adjustment. However, since all markets are not efficient or effective in this way, the ERC model, shown here as equation (2), is widely used to describe the cause and effect of these deviations (Collins & Kothari 1989).

(2) $CAR_{ir} = \gamma + (ERC_i)\,(UE_{ir}) + e_{ir}$

where:

CAR_{ir} = cumulative abnormal returns on stock i for interim report r,
γ = intercept,
ERC_i = earnings response coefficient (slope of CAR on stock i),
UE_{ir} = unexpected earnings of stock i for report r, and
e_{ir} = n.i.d. errors of CAR on stock i for report r.

The very existence of equation (2) assumes that in certain conditions, the expected value of CAR_{ir} is not equal to zero. Equation (2) assumes that deviations from the central location of CAR_{ir}, whatever value the CAR_{ir}'s may be, are centered on zero and have a constant, bell-shaped distribution.

The Vaga/Martin theory describes a condition in which e_{ir} may not be centered at zero and/or may not have a constant, bell-shaped distribution. This article argues that such conditions sometimes exist. Specifically, the distribution of e_{ir} is assumed to be described by the generalized density function shown in equation (3).

(3) $\text{Log}(f[x]) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$,

where θ_0 is a normalizing constant. With a suitable reparameterization we may always take $\theta_3 = 0$. In order to have a proper distribution the parameter θ_4 must be nonpositive. The distribution family includes the normal distribution, which occurs when $\theta_2 < 0$ and $\theta_3 = \theta_4 = 0$. The result is then a parabolic log-likelihood function with a unique maximum. With θ_4 negative, a wider family of distributions including even skewed and bimodal distributions is obtained. The θ_4 coefficient specifies the unit of measurement and therefore it can be chosen arbitrarily, here $\theta_4 = -1/4$. In this case the log-likelihood function is a fourth order polynomial which may possess two maxima. The derivatives of equation (4) below are polynomials of degree three. If this polynomial has only one real root solution, the distribution is unimodal. If it has three roots, there are two peaks with one valley between them. Thus, the error term (e_{ir}) may not have a unique mode. This corresponds to the case when conflicting signals are provided to the market place. The sign of Cardan's discriminant

(4) $CD = (\theta_1/2)^2 - (\theta_2)^3$

indicates whether the number of modes is one or two. The parameter θ_1 can be interpreted as a skewness parameter and θ_2 as a kurtosis parameter.

In bimodal cases, the density function has three critical points: two maxima and one minimum. The minimum is essentially an unstable equilibrium. The two maxima represent two stable equilibria. The location of the distribution is usually measured by the mean, which is located somewhere between the two maxima but is usually not the local minimum. In mean square sense, the mean is the best single point to use in describing the location of the distribution. Yet, in likelihood sense, either of the maxima is a superior measure of location.

Astute investors realize that the initial pricing is biased, but they do not know the direction of the bias. In practice, the initial market response to bimodal information is its mean value. The mean is in the neighborhood of the minimum point. Since the mean is close to the trough of the distribution, an initial market return that is located near the mean is unstable. Subsequently, a more correct consensus will develop around one or the other of the two maxima. In the beginning of a reaction period, the user of the information knows that the return that best reflects the inherent value is located at one or the other of the maxima. However, due to the inability to determine which one of the two it actually is, the minimum risk position is the average point. Thus, the initial return, in a situation with mixed signals, is: (1) lower than the potential maximum expected return (located at the upper lobe) and (2) higher than the potential minimum expected return (located at the lower lobe). Due to the inherent instability of the bimodal case, a consensus is quickly reached. Other research indicates this to be within a ten day period on the Helsinki Stock Exchange (HSE) (Schadewitz & Blevins 1997b).

In this research, bimodality is modeled in the error distribution of the ERC model that is presented in equation (2). The test is conducted on the release date of most interim reports submitted to the HSE over the period 1985–93. The most natural causes of unexpected favorable or unfavorable information are the quantity and the quality of the items disclosed in those reports. Thus, this research assumes that shape parameters for the log-likelihood density functions of equation (3) are determined by interim disclosures. Equation set (5) states the relationships assumed for each of the parameters:

$$(5) \quad \begin{aligned} \theta_1 &= a_0 + a_1 di + a_2 di^2 \\ \theta_2 &= c_0 + c_1 di + c_2 di^2 \\ \theta_3 &= 0 \\ \theta_4 &= -1/4. \end{aligned}$$

The element "di" refers to a disclosure index, that is briefly discussed in the section entitled "Data." Combining the first five equations results in equation (6):

$$(6) \quad l(\alpha,\beta,\theta_0,a_0,a_l,a_2,c_0,c_1,c_2) = \Sigma_i\ [\theta_0 + (a_0+a_l di+a_2 di^2)x_i + (c_0+c_l di+c_2 di^2)x_i^2 - x_i^4/4]$$

where θ_0 is a normalizing constant. This highly nonlinear function is then maximized with respect to its parameters. The variances of the maximum likelihood estimators are obtained using the inverse of the information matrix. The maximization is carried out by a Gauss program kindly provided by Vance Martin (Martin 1990).

DATA

The sample is based on practically all the interim reports published by HSE listed firms during the period 1985–93. The finance and insurance sectors are excluded, due to their intertemporal and cross-sectional variability (Niskanen 1990). At this stage, there are 573 interim reports remaining. Outliers and reports with missing data points are next eliminated. This leaves a sample size of 491 usable interim reports.

Calendar year 1985 is selected as the starting year for this research because: (1) the systematic filing of interim reports to the HSE began in 1985 and (2) the interim reports published prior to 1985 are no longer available.

The instrument used in the data collection is a scoresheet, which summarizes the information contained in the original 573 interim reports. Scoring follows the Giner Inchausti (1993) methodology, with the addition of an option that identifies a data element that is not reportable. A complete list of the items in the index is provided in Kanto & Schadewitz (1997). The disclosure index includes all of the elements that are most commonly discussed in the literature (Schadewitz & Blevins 1997a). This list is consistent with those published by the U.S. based Association for Investment Management and Research (AIMR 1992). The list is also confirmed by interviews with Finnish professional practitioners. The index for any report is the proportion of weight it possesses to the weight it would possess if all reportable items were maximized. Since the maximum total weight differs from one report to another, each index number is interim report-specific. A detailed description of this, including a complete discussion of the data sources is found in Schadewitz & Blevins (1998). The examination of each interim report is conducted on the date of its release.

RESULTS

The total number of firms in the data set is 62, with 27 of these (43 percent) displaying bimodality. The total number of usable interim reports is 491, with 77 (15 percent) exhibiting bimodality. Bimodality is identified mechanically, *via* equation (6). Confirmation of the efficacy of the model is accomplished subjectively. Table 1 reports the seven most pronounced cases. For a more detailed analysis, see Appendix.

The core reasons for market behavior that appear to be imbedded in these interim reports are categorized as: (1) favorable and (2) unfavorable information, at least some of which is unexpected.

IMPLICATIONS

In the context of this research, bimodality occurs when an interim report contains some information that is unexpectedly favorable and other information that is unfavorable, with some of the information being new. Application of the market model develops a bimodal residual distribution. One of the two equilibria is positive. The other is negative. This explains why the regression residual of market models are sometimes: (1) not centered at zero and (2) not statistically significant.

The well known adjustment lag is the time it takes for the market to digest the newly published information. In a bimodal case, the lag is exacerbated by the time it takes for the market to move from the unstable mean to one or the other of the peaks. The new insight

TABLE 1. Market response to interim report information

Firm	Bearish information	Bullish information
Oy Wärtsilä Ab	Collapse of Soviet Union	Superior results compared to the previous year
Metra Group	A slump in demand in main markets	Investments in foreign firms
Tampella Group	Price development of carboard	A large investment program
Outokumpu Oy	Several unsatisfactory segments	Steel segment satisfactory
Effjohn Oy Ab	Decrease in the number passengers	Merger with overlapping businesses
Amer Group Ltd	Decreased demand for brand products	Large mergers into new branches
Tietotehdas Oy	Uncertain development of computer business	A large acquisition

provided by this research is that the variability in adjustment lags might be a function of the time required for the market to search for the true equilibrium.

All these cases resemble figure 1 and illustrate that the returns associated with one of the peaks are slightly positive, whereas the returns associated with the other maximum are severely negative. Further, the positive peak has a much greater volume. Thus, in the illustrative cases, the vast majority of investors view of the future prospects as slightly favorable; whereas, a small minority of investors expect a very bad future.

CONCLUSIONS

The received market model assumes the error term to be centered at zero, with a constant bell-shaped distribution. Standard regression studies have long recognized that the center of the disribution of abnormal returns is usually *not* zero. Further, the tails of the t distribution, which is used to assess the residual, are so thick that the levels of confidence associated with such studies are often disappointingly low.

In addition, the reported adjustment lag appears to be widely variable. Historically, the adjustment lag literature has not been tied closely to the residual location/dispersion literature. There may be a very close connection that has been overlooked.

This research argues that there may be a single, sensible explanation for all three of these phenomena. The explanation is that, in some cases, the unimodal residual assumption, that is associated with the market model, is fallacious. The theoretical work done by Vaga (1990) and the methodological work done by Martin (1990) provide the basis for this research. The argument is that, when the unexpected information received by the market contains both favorable and unfavorable indicators, and some of this material is new, a bimodal distribution of abnormal returns is generated. Because one or the other of the maxima often contains a greater density than does the other, the mean of the distribution is not always zero. That explains the mean unequal to zero problem. The very existence of a bimodal distribution suggests a wider variance than that which would be expected with a t distribution. That explains the reason why some levels of confidence are disappointingly low. The fact that the bimodal condition is transitory implies that some time will elapse before a stable consensus is reached. Since the information underlying the distributions differs, there is ample reason for varying adjustment lags.

To test this argument, a theoretical model is developed *via* the Vaga/Martin framework. This model focuses on the market response to interim reports. The theory is tested using data collected from interim reports submitted to the HSE over the period 1985–93. The model mechanically identifies firms that submit interim reports that cause bimodal return distributions. Subjective analysis of the interim reports confirms the existence of bipolar information.

Although the model reported here is tested using interim reports submitted to the HSE from 1985–93, the argument has implications for the assimilation of mixed information, conveyed by any media to any forum over any period of time. ■

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APPENDIX

(1) Oy Wärtsilä Ab: stationary engines and turbines manufacturer

Double peaks – 1989–90.

Example of conflicting information – Interim report(s) covering the period: January – April, 1990.

Favorable indicator(s) – The increase of net sales has been strong. This development has been accelerated due to newly merged companies. All the industry-lines report superior results compared to the previous year (January – April, 1989).

Unfavorable indicator(s) – The former administration of Wärtsilä has been sued by shareholders, who are seeking to receive compensation related to: (1) share issues and (2) order backlog transfers. Further, Wärtsilä has large contracts with the Soviet Union to supply new icebreakers. The collapse of the USSR causes the interim reports to present very unfavorable prospects for the firm.

(2) Metra Group: the heavy metal and building industry

Double peaks – 1991–93.

Example of conflicting information – Interim report(s) covering the period: March – August 1991.

Favorable indicator(s) – In certain cases, the profitability in those divisions with a more developed international presence actually exceeded targets. International acquisitions objectives were met last year.

Unfavorable indicator(s) – ”When the Metra Group was planning its operations for the current year, there were clear signs of a slump in demand in the main markets. However, the recession has turned out to be far more severe than expected, particularly in Finland, and the outlook is fraught with a number of unpredictable factors.” and ”The Metra Group failed to achieve its sales and profit targets during the review period. The largest shortfalls were experienced in the divisions most dependent on Finland and the other Nordic countries.”

(3) Tampella Group: pulp and paper making machine manufacturer

Double peaks – 1988–93.

Example of conflicting information – Interim report(s) covering the period: January – August 1988.

Favorable indicator(s) – A large investment program was introduced. Furthermore, the overall development and future prospects of business lines were favorable. There is also a column indicating Tampella’s strong initiative to internationalize its operations *via* acquisitions.

Unfavorable indicator(s) – Prices in the cardboard industry are unsatisfactory.

(4) Outokumpu Oy: iron and steel manufacturing

Double peaks – 1989–93.

Example of conflicting information – Interim report(s) covering the period: January – April 1991.

Favorable indicator(s) – The results of the steel segment has been satisfactory.

Unfavorable indicator(s) – The results of all other segments have been unsatisfactory.

(5) Effjohn Oy Ab: sea transport of passengers

Double peaks – 1990–93.

Example of conflicting information – Interim report(s) covering the period: January – August 1990.

Favorable indicator(s) – Interim report January – August 1990 discloses that Effoa and Johnson Lines are merged. Previous literature indicates that the merger of firms with overlapping lines should be favored by its shareholders (Healy, Palepu & Ruback 1992). In year 1993, interim reports contain information that the programs to increase the efficiency of the firm have been successful. The amount of passengers have increased and is expected to increase in the future.

Unfavorable indicator(s) – A large portion of its income comes from personal traffic. The economic recession in Finland causes people to reduce their spending on leisure traveling. The interim report explicitly mentions overcapacity available on the market. Yet, the firm is waiting for a new, large capacity vessel to be ready for its route.

(6) Amer Group Ltd: tobacco products manufacturer

Double peaks – March – August 1989.

Example of conflicting information – Interim report(s) covering the period: March – August 1989.

Favorable indicator(s) – Large mergers into new branches. Expansion of brands into its product portfolio. Stock purchase of Wilson Sporting Goods Co. (makes the operations of Amer Group global, as well as more production oriented. Among other things, it is said that Wilson will be an important corner stone and that it should help to diversify risk). Crucial extension towards leisure consumer goods that are close to consumers – New information.

Unfavorable indicator(s) – Economic recession, Bearish expectations stock market – Old information.

(7) Tietotehdas Oy: computer and data processing services

Double peaks – October 1987 – March 1988.

Example of conflicting information – Interim report(s) covering the period: October 1987 – March 1988.

Favorable indicator(s) – Tietotehdas reports that it has acquired a majority in Datema. At that time, Datema is the largest data-service and data-programming firm in the Nordic countries. According to the interim report, Tietotehdas gaines a strong market position in Sweden as well as a foothold to Norway and Denmark through the acquisition.

Unfavorable indicator(s) – The future of computer business as a whole is uncertain at the time.