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## **Estimating A Smooth Term Structure of Interest Rates**

The term structure of interest rates measures the relationship, at a given point of time, between time to maturity and the yield to maturity of default-free zero-coupon bonds. This functional relationship is the key concept in the fixed-income market. For example, it is needed in the active fixed income portfolio management, the value-at-risk systems requires a good tool to map the cash flows from fixed income instruments in order to estimate the risks in the portfolios, and pricing of instruments demands a zero-coupon yield curve estimate.

The zero-coupon yield curve can be represented in any of the following ways: the discount function, the zero-coupon interest rates, or the forward rates. Usually, it is illustrated by a zero-coupon curve. Unfortunately, there is a problem to determine the zero-coupon curve, as zero-coupon bonds with maturities over a year are seldomly traded. However, the term structure of interest rates can be estimated from coupon bonds, which can be viewed as a package of zero-coupon instruments.

Although the estimation of the yield curve is in theory pretty easy there are several empirical problems that can seriously affect the goodness of the fitted yield curve. First, the number of coupon bonds can be small. Second, the shape of the yield curve must be allowed to vary. Third, market imperfections can seriously affect the fitting method.

This paper proposes a new spline-based method to estimate a smooth term structure of interest rates. The proposed method extends the literature by using a different smoothing norm, the square of the discontinuity jump in the third derivatives at the interior knot points, and by locating the internal knot points by the size of the fitting errors. The proposed method also uses the generalized cross validation to detect the optimal smoothness of the curve.

The method is applied to the Finnish fixed income market, where the number of instruments is very small compared to the cash flow dates. The term structure of interest rates is estimated using six money market instruments and five government benchmark bonds during the period June 3, 1993 through February 6, 1996. The new method is compared to other smoothing spline methods and to the equally spaced knot position method. The results show that the new method outperforms the older ones when the spline is placed on the log of the discount function and when the knots are located by the size of the fitting errors.