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Demand Uncertainty in a Cournot-duopoly

1. INTRODUCTION

In a standard duopoly setting the firms choose either prices or quantities in a non-cooperative fashion. In the jargon the former is Bertrand-competition and the latter Cournot-competition. It may be thought that both firms choose simultaneously, and if the firms are symmetric, a symmetric equilibrium is usually studied. Sometimes one of the firms is given the opportunity to make its decision before the other firm. In a Cournot-setting the situation is labelled Stackelberg-competition. Apart from the general interest of studying this case it is usually motivated by the first mover being a large firm, a market leader, or possessing superior information.

We endogenise the decision to move first or second, and assume that the competitive phase consists of two periods. In either period the firms can make a production decision that is irreversible. As far as the firms are allowed to choose (non-cooperatively) the period they make the decision, one can study the circumstances that favour sequential rather than simultaneous decisions. If this is the only change in the standard setting with perfect information there are now three pure strategy subgame perfect equilibria when the firms are symmetric (see Hamilton and Slutsky, 1990). Either of the firms is a leader and the other one a follower, or both of them make the same choices as in the standard setting in the first period. The firms prefer the equilibrium in which they move first. Even though no equilibrium selection is helpful here the symmetry of the situation makes the symmetric equilibrium appealing.

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In our model the production period plays a non-trivial role since we assume that the demand is uncertain, and that the uncertainty is resolved once either firm makes its production decision. The enterprise bears a close relationship to the literature about endogenous timing of moves in oligopolies (Canoy and Cayseele, 1996; Gal-Or, 1987; Kulatilaka and Perotti,1991; Mailath, 1993; Norman, 1996; Albaek, 1990; Hamilton and Slutsky, 1990). With the exception of the last one these articles deviate from the standard model by giving up perfect information. Uncertainty as such, however, is not sufficient to generate endogenous timing of moves or equilibria with only sequential decisions. Some asymmetries between the firms have to be introduced, too. In Gal-Or (1987), Mailath (1993), and Norman (1996) one of the firms may have better information about demand than others which creates an asymmetry between firms. This turns the model into a signalling game which has notoriously many equilibria and may be rather involved to solve.

The articles of Albaek (1990), and Hamilton and Slutsky (1990) are closest to our work. In the former there is uncertainty about the firms' costs. Neither firm knows the other one's production costs but they know the expected values and variances of the costs. With linear demand this is sufficient to guarantee that for some parameter values there exists a so called *Natural Stackelberg situation* in which one firm prefers being a leader to being a follower and to choosing simultaneously, and the other firm prefers being a follower to being a leader and to choosing simultaneously. This requires that the variances differ sufficiently, and the firm with the greater variance is the leader. In our model simultaneous moves is never an equilibrium, and depending on the variance either the first mover or the second mover may be more profitable.

Hamilton and Slutsky (1990) study two different games: A game in which firms announce in which period they are going to produce, and are committed to this announcement, and a game in which the firms can choose in which period to produce only by actually producing. Our model corresponds to the latter game the difference being uncertainty about demand.

The crucial assumption in our model is the way demand is revealed. If either firm produces in the first period demand is known in the second period. In case neither firm produces in the first period demand remains unknown in the second period. This is clearly a very specific assumption that applies only to some markets. Perhaps, the most important is the case of new products. Demand for new products is highly uncertain, and many times the only way to find it out is to enter the markets by producing the product. A case of low consumption light bulbs is cited in Mora (1995); they were evidently not produced in the beginning of the 1970' because of demand uncertainty which was endogenous in the sense that since no-one produced there was no way to update the priors about demand. The new-product situation applies in an evident way to locations, too. For instance, Lafontaine cites examples of fast food chains where one chain's new outlet is frequently followed by an outlet of another chain.

Canoy and Cayseele (1996) study a price setting game between differentiated firms, which is not directly relevant to our quantity setting framework. However, they have an example about tour-operators; Sunair-Airtour, in Belgium always provides its catalogue of winter holidays much before the others. This clearly leaves them open to the threat of other operators undercutting their prices, and this is the point that is emphasised in Canoy and Cayseele (1996). We look at the matter from a different perspective of demand uncertainty. In many businesses demand varies a lot, and production decisions have to be made well before the realisation of demand. Committing to quantity in such an environment very early gives a firm a first mover advantage corresponding to a Stackelberg-situation. The trade-off comes from choosing the wrong quantity with respect to the realised or true demand. Kulatilaka and Perotti (1991) resembles our model as to the timing. A firm that makes its investment decision early attains lower unit costs than a firm that postpones its investment decision. Waiting entails the advantage of being fully informed about demand. We take the firms' costs as fixed and concentrate on demand uncertainty exclusively.

Similarly in the present model the assumption that demand is revealed if and only if at least one firm makes a production decision in the first period after which it cannot change its production in the second period is open to criticism. Presumably, the demand is revealed since some products are sold in the first period, and then a question arises why the firm cannot produce more in the second period if demand turns out to be strong. The answer is the same as in the standard Stackelberg -case; it is assumed that the firms are committed to the levels of production which they choose. The standard static case is an approximation of a dynamic real life situation that is compressed into two stages. Our model can be regarded as an approximation of a real life situation in which a producer brings a new product to the market. First he has to expend his time in production, and only after this he sells or markets the product which is time consuming as well. The competitor produces while the first producer sells his products. Compressing this idea into a two stage model makes analysis tractable, and captures the idea that there is a trade-off between producing early, and being well informed about demand. As a side product of this kind of modelling we get an interesting result that simultaneous moves equilibria vanish.

We choose this particular timing structure and revelation of demand since we think that it is relevant especially to the case of new products. However, one can also think of the following case. There are two periods or stages when the firms can make production decisions. In the first stage demand is unknown like in our model. In the second period is revealed even in the case that both firms decide not to produce in the first period. The results in this case are slightly different; when uncertainty about demand is not too great the firms move sequentially in equilibrium. If the demand uncertainty is large both firms make their production decision in the second period when demand in known for certain.

In section 2 we present the model and the results, and in section 3 we present the conclusions.

2. THE MODEL

There are two firms and two time periods or stages. Both firms produce a homogenous good. Demand is linear $x = a - p$, where p is price, x is total quantity demanded and *a* is the realisation of a random variable $A \in [\underline{a}, \bar{a}]$ which is assumed to have a continuous density. In the first period *a* is unknown. The density of *A* is, however, common knowledge. The expected value of a is a^*. If q_1 and q_2 are the output levels of the two firms, the inverse demand is $a - q_1 - q_2$. We assume that the variance of *A* is not too large in a sense that in no case the firms produce so much that price drops to zero. The two firms have a marginal cost of production *c* and no fixed costs.

We want to study the effect of information revelation on the timing of the firms' production decisions. To this end we model the firms playing the following extensive form game. The firms make their decisions non-cooperatively, and they may choose the quantity to be produced in either period. If a firm produces already in period one the choice is common knowledge in period two, and the true demand is revealed. If neither firm produces in the first period no information about demand is revealed in the second period. Notice that the game is not a signalling game, and that unlike in many models only actions speak; firms commit to a production decision by producing, not making announcements about when they intend to produce and how much (Albaek,1990; Hamilton and Slutsky, 1990). Next we determine the profits when the firms move sequentially and simultaneously, and then we compare the profits in the two cases.

Sequential decisions

Without loss of generality let firm 1 be the first mover and make its decision in the first period. Firm 2 is the follower which delays its production decision until the second period. Firm 1 believes (correctly) that its production decision in period one will influence firm 2's decision a period later. That is, the follower will select q_2 to maximize its profits

(1) $$\max \pi_2 = [a - q_1 - q_2(a) - c]\, q_2(a)$$

The first order condition is

$$\frac{\partial \pi_2}{\partial q_2} = a - q_1 - 2q_2 - c = 0$$

from which one gets

(2) $q_2(a) = \frac{1}{2}(a - q_1 - c)$

First mover's decision problem is to maximise expected profit

(3) $\max E(\pi_1) = E[a - q_1 - \frac{1}{2}(a - q_1 - c) - c]\, q_1 = \frac{1}{2}(a^* - q_1 - c)\, q_1$

with first order condition

$$\frac{\partial E(\pi_1)}{\partial q_1} = \frac{1}{2}(a^* - 2q_1 - c) = 0$$

from which one gets the quantity

(4) $q_1 = \frac{1}{2}(a^* - c)$

Using (4) the follower's choice (2) can be rewritten

(5) $q_2(a) = \frac{1}{4}(2a - a^* - c)$

and the equilibrium price turns out

(6) $p(a) = a - q_1 - q_2 = \frac{1}{4}(2a - a^* + 3c)$

First mover's expected profit is

(7) $E(\pi_1) = E[(p(a) - c)\, q_1] = E[\frac{1}{4}(2a - a^* - c) \times \frac{1}{2}(a^* - c)] = \frac{1}{8}(a^* - c)^2$

while the follower gets in expected terms

(8) $E(\pi_2) = E[(p(a) - c)q_2] = E[\frac{1}{4}(2a - a^* - c) \times \frac{1}{4}(2a - a^* - c)] =$
$\frac{1}{16}E[4a^2 + (a^*)^2 - 4aa^* + 2a^*c - 4ac + c^2] =$
$\frac{1}{16}(a^* - c)^2 + \frac{1}{4}E(a^2 - ac - aa^* + a^*c) =$
$\frac{1}{16}(a^* - c)^2 + \frac{1}{4}E(a^2) - (a^*)^2] = \frac{1}{16}(a^* - c)^2 + \frac{1}{4}\mathrm{var}(a)$

First mover has an advantage if $\frac{1}{8}(a^* - c)^2 > \frac{1}{16}(a^* - c)^2 + \frac{1}{4}\text{var}(a)$ which is equivalent to

(9) $(a^* - c)^2 > 4\ \text{var}(a)$

If variance in a is small the usual Stackelberg-case where the first mover has always an advantage prevails. Only if the variance is large the first mover may fare worse than the second mover. Note that the variance does not show in the first mover's profit. This comes from the linear demand, and the fact that the variance is assumed small enough so that realised prices are always positive. The second mover's profit has a variance terms since variance indicates the pay-off from waiting as the second mover knows the demand exactly.

Simultaneous decisions

As long as both firms make their production decisions simultaneously the profits are the same regardless of the period since the assumptions about the revelation of information guarantee that the demand is unknown. Firm 1 maximises its expected profits

(10) $\max E(\pi_1) = E[a - q_1 - q_2 - c]\, q_1$

From the first order condition we get

(11) $q_1 = \frac{1}{2}(a^* - q_2 - c)$

Since the firms are identical, firm 2's production is symmetric

(12) $q_2 = \frac{1}{2}(a^* - q_1 - c)$

and in equilibrium output decisions are the same

(13) $q_1 = q_2 = \frac{1}{3}(a^* - c)$

and the price is

(14) $p(a) = a - q_1 - q_2 = \frac{1}{3}(3a - 2a^* + 2c)$

Firm 1's expected profits are

(15) $E(\pi_1) = E[(p(a) - c)q_1] = E[\frac{1}{3}(3a - 2a^* - c) \times \frac{1}{3}(a^* - c)] = \frac{1}{9}(a^* - c)^2$

which is the same as firm 2's expected profits

(16) $E(\pi_2) = \frac{1}{9}(a^* - c)^2$

It is easy to establish that in equilibrium both firms do not produce in the first period; a revealed preference argument is sufficient to establish this. Assume that there is an equilibrium in which both firms produce in the first period. Denote the firms' equilibrium productions by q_1^* and q_2^*. Consider, say, firm 1. If it deviates and waits until the next period when it gets to know the demand. It can still produce q_1^* but with full knowledge of the demand this production level is not the optimal choice. Firm 2 produces $\frac{1}{3}(a^* - c)$ and the upcoming production of deviating firm 1 will be $\frac{1}{6}(3a - a^* - 2c)$, whereas the price will be $\frac{1}{6}(3a - a^* + 4c)$. The expected profits are $\frac{1}{9}(a^* - c)^2 + \frac{1}{4}var(a)$; $\frac{1}{4}var(a)$ higher than if the firm would not deviate. This shows that there is no equilibrium with both firms producing in the first period. Thus, there are three possible equilibria: Firm 1 produces in the first period and firm 2 in the second period, firm 2 produces in the first period and firm 1 in the second, and both firms produce in the second period.

Next we compare the profits in the sequential, and simultaneous moves cases to determine whether or when sequential moves are more profitable than simultaneous moves. Whenever the first mover's expected profit is larger than his expected profit in the simultaneous move case, simultaneous moves is not an equilibrium. But from (7) and (16) we see that this is always the case. We must still show that firm 2 does not deviate and produce in the first period, when firm 1 is already producing $\frac{1}{2}(a^* - c)$ in the first period. Firm 2's optimal production choice in the first period is $\frac{1}{4}(a^* - c)$. The future price level will be $a - \frac{3}{4}(a^* - c)$ and firm 2's expected profits are $\frac{1}{16}(a^* - c)^2$; $\frac{1}{4}var(a)$ less than if the firm waited to the next period. Thus, firm 2 does not deviate and there does not exist an equilibrium in which the firms move simultaneously.

We want yet compare the expected profits of the two moving alternatives. Firm 2 prefers sequential solution to simultaneous moves if (8) is larger than (15), i.e. if $\frac{1}{16}(a^* - c)^2 + \frac{1}{4}var(a) > \frac{1}{9}(a^* - c)^2$ which is equivalent to

(17) $36\, var(a) > 7\,(a^* - c)^2$

The previous condition (9) that each firm would rather be the first mover was $(a^* - c)^2 > 4\, var(a)$. We can combine these two conditions

(18) $(a^* - c)^2 > 4\, var(a) > \frac{7}{9}(a^* - c)^2$

If (18) holds the first mover earns more than the follower and both firms prefer sequential moves to simultaneous moves. We summarise these results as

Proposition 1. With demand uncertainty there are exactly two equilibria in both of which the firms move sequentially.

Proposition 2. For large enough variance, i.e. if $(a^* - c)^2 < 4\ \mathrm{var}(a)$ the second mover earns higher profits than the first mover. Otherwise the first mover earns higher profits.

These results are quite sharp. In cases where demand uncertainty is revealed only after at least one firm produces there are no simultaneous moves equilibria. The case in which both firms move simultaneously in the first period is not an equilibrium since either firm can wait till the next period when it has the same choice set as in the first period, and additionally it knows the demand exactly. The case in which both firms move simultaneously in the second period is not an equilibrium roughly because a deviating firm gains a first mover advantage. Generally this is an advantage only with respect to the simultaneous moves case since it is possible that the second mover's profits are greater than the first mover's profits. The greater the uncertainty the more likely it is that the second mover has an advantage over the first mover profitwise.

3. CONCLUSIONS

We have shown that introduction of uncertainty about demand endogenises the timing of moves in a quantity setting duopoly in a sense that only the asymmetric equilibria survive. The heuristics behind the result is that the first mover gets the advantage that is familiar from the Stackelberg-setting, while the second mover makes the move endowed with better information than the first mover. In the undeniably special setting the first mover does not know the demand and if there has been a first mover the second mover has better knowledge about the demand. In case both firms decide to wait till second period before moving no information is revealed. The pay-off from these assumptions comes in that the game between the firms is not a signalling game which usually possess a huge number of equilibria, which in turn makes clear-cut results rare. Our model is clearly inappropriate in some areas of business but it may be close to truth in other areas; introduction of new products, establishing outlets in new locations, and cases like holiday travel business where the decisions have to be made well before demand is fully revealed.

In the model the firms are symmetric in all respects and it is not possible to say which firm will be the first and second mover. When the firms differ in costs the calculations (available from the authors) reveal that the firm with lower costs is more likely to be the first mover. The heuristics is that for the high cost firm incorrect choices made under imperfect information are more costly than for the low cost firm. Thus, the high cost firm prefers to wait for the resolution of demand uncertainty, and lets the low cost firm to take 'the first mover advantage'.

REFERENCES

ALBAEK S. 1990, Stackelberg leadership as a natural solution under cost uncertainty, Journal of Industrial Economics 38, 335–347.

CANOY M. AND P. VAN CAYSEELE 1996, Endogenous price leadership with a front-runner, manuscript presented in EARIE 1996 Vienna.

GAL-OR E. 1987, First mover disadvantages with private information, Review of Economic Studies 54, 279–292.

HAMILTON J. H. AND SLUTSKY S. M. 1990, Endogenous timing in duopoly games: Stackelberg and Cournot equilibria, Games and Economic Behavior 2, 29–46.

KULATILAKA N. AND PEROTTI E. 1991, What is Lost by Waiting to Invest ? A theory of Strategic Real Options, Manuscript .

MAILATH G. 1993, Endogenous sequencing of firm decisions, Journal of Economic Theory 59, 169–182.

MORA J.V.R. 1995, Shared knowledge, Economics Working Paper 144, Universitat Pompeu Fabra.

NORMAN H-T. 1996, Endogenous timing in a duopoly model with incomplete information, EU Working Papers No. 96/20, European University Institute.