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# A Simulation Assessment of Three Methods for Deriving the Long-run Profitability of the Firm as its Internal Rate of Return*

**ABSTRACT**

*Three long-term profitability estimation methods, Kay's, Ijiri's and the average accountant's rate of return method are evaluated using simulated financial statements. It is observed that the methods are disrupted by large deviations between growth and profitability, but are insensitive to cyclical fluctuations. The numerical performance of the methods is roughly at par, but Ijiri's method is more unpredictable than the theoretically better founded Kay's method. The average accountant's rate of return method fares almost as well as Kay's method, and can be recommended for financial analysis because it is based on well-established accounting practice.*

**Key words:** *Long-term profitability, accountant's rate of return, internal rate of return, IRR estimation models, simulation.*

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# 1. INTRODUCTION

## 1.1 Background

Measuring the firm's profitability is a central task both in accounting practice and theory. The stakeholders of the firm need the profitability information for their decision making both in the short and in the long run. In the economics literature the internal rate of return (IRR) is the widely used theoretical long-run profitability concept. A recent survey by Pike (1996) in the area of capital budgeting confirms that IRR is a well-established measure also among practitioners. Furthermore, the investment theory of finance recognizes IRR as a profitability measure, albeit under restrictive assumptions.

Strictly speaking finance theory states that for example under capital rationing only the net present value method is uniquely consistent with maximizing the value of the stockholders' wealth. See any good text-book of finance such as Copeland and Weston (1979), Levy and Sarnat (1986), Brealey and Myers (1991) for a discussion. However, under ordinary practical conditions of investment opportunities in the same size categories and conventional cash-flow patterns the internal rate of return method can in most cases be expected to give conforming evaluation for the capital investment evaluation. In this paper we take IRR as the true long-run profitability measure for the firm.

Applying the internal rate of return as a measure of the ex-post long-term profitability of the firm is not straight-forward in actual practice. Since the mid 1960's there is a long-standing controversy, both conceptual and technical, whether it is possible successfully to estimate the firm's IRR. See the review article by Salmi and Martikainen (1994; Section 3) and Butler, Holland and Tippett (1994), and Stark (1994) for the references.

The approaches in literature to the IRR estimation can be classified into several, partly overlapping categories. One approach has been trying to establish a correspondence between IRR and ARR (the accountant's rate of return). This approach is exemplified by Kay (1976) and later by Peasnell (1982a, 1982b) and Steele (1986). Kay's method has been evaluated for example by Salmi and Luoma (1981), Brief and Lawson (1992) and Salmi and Virtanen (1995a). Another approach is to derive the IRR by utilizing an auxiliary estimate such as CRR (the cash recovery rate). This approach has been suggested by Ijiri (1979 and 1980), extended and tested by Salamon (1982) and Gordon and Hamer (1988). Ijiri's method has been further tested by Shinnar, Dressler, Feng and Avidan (1989) and Stark, Thomas and Watson (1992). Lawson (1980) presented an approach based on cash flows and market values. Furthermore, there are approaches seeking to estimate the IRR directly from the published financial statements. This category is represented by Ruuhela (1972) and the mathematically streamlined rederivation of Ruuhela's model in Salmi (1982).

Which of the various methods put forward in literature should one select? For the business practitioner, as well as for an academic researcher, facing the number of the various long-run profitability estimation methods, and the theoretical controversy of their validity, the question becomes the following. What method is applicable for evaluating the long-run profitability of a business firm? In other words which method or methods work both in practice and in theory?

## 1.2 Research Problem and Methodology

This paper evaluates the estimates given by Kay's and Ijiri's methods and compares them to the simple practice of using the average of the annual accountant's rate of returns as the estimate of the firm's IRR. The evaluation of the cash flow and market value based methods and the direct methods remain the subject of further research.

The long-term profitability IRR estimation methods in literature all are mathematically non-trivial. They are not straight-forward to apply in practice on actual financial data. The practitioner's obvious alternative would be to use the average accountant's rate of return as a surrogate of the IRR estimate. Doubts on this idea can be traced as far back as to Vatter (1966). Later e.g. Fisher and McGovan (1983:82) stated that "accounting rates of return provide almost no information about economic rates of return". On the other hand, as pointed out by Pike (1996; 83–84) in connection with capital budgeting, the technically simple methods such as the payback period and the average ARR has been condoned by several authors starting from Weingartner (1969).

We intend to revisit the question of the usefulness of the average ARR as an ex-post long-term profitability measure, since it has not been unequivocally demonstrated that the average ARR method would necessarily be markedly inferior to the more complicated IRR estimation methods presented in literature. Hence we will consider the average ARR method together with Kay's and Ijiri's methods in this paper. Two issues arise based on the earlier doubts on the average ARR method. First, is it a fair estimate of the true IRR. Our results indicate that the answer depends above all on the relationship between the firms growth and profitability. Second, will the average ARR do markedly worse that Kay's and Ijiri's methods. Our results do not support the part of the literature rejecting the usefulness of the ARR as the IRR substitute.

The established convention in the long-run profitability research like Salamon (1982; 294) is to consider the firm as a series of repetitive capital investments. Stating this research convention in Salamon's words "... the firm is a collection of projects that have the same useful life, same cash-flow pattern, and same IRR". See, however, the critique of this standard assumption by Kelly and Tippet (1991).

This approach implies the strong assumptions about the firm's access to the financial markets to freely obtain the funding for the capital investments. In other words the implied capital markets in this area of research conventionally are perfect and complete. There is no capital rationing. Therefore, the financing of the capital investments need not be considered in this paper.

In the current paper we are interested in the accuracy of the selected profitability estimation methods under different economic circumstances, investment strategies and accounting decisions. More specifically, the following research questions will be considered.

In the earlier research a constant growth approach to the capital investments has been fairly common. We relax this restriction. Therefore, our first research question is

1) Are the methods sensitive to cyclical fluctuations in the capital investment activities?

Second, an outside stakeholder has to base the profitability estimates on the financial data provided by the firm. In the financial statement data the capital investments and their cash flows are totally mixed. It is not possible to know the contribution pattern of the capital investments based on the external data. The question of the effect of the different contribution patterns arises as in Salamon (1982) and Gordon and Hamer (1988). Hence, our second research question is

2) Are the methods sensitive to the underlying, assumed cash contribution patterns and duration of the firm's capital investments?

Third, it has been put forward in the earlier literature that there are some particular instances where the profitability estimates theoretically become close or equivalent to the underlying, true profitability of the capital investments making up the firm. These include the case where growth equals profitability as presented by Solomon (1966; 115) and the case where the theoretical annuity method of depreciation is postulated as presented in e.g. Salmi and Luoma (1981; 28) and Peasnell (1982a; 364). The annuity depreciation is the economist's depreciation in defining the concept of economic income discussed e.g. in Bromwich (1992; 31–51). Hence, our third research question is

3) Are the methods sensitive to disparities between the firms growth and profitability?

Fourth, in accounting practice the choice between the depreciation methods such as the prevalent straight-line and the declining balance methods affects the reported annual income figure. Our fourth question is

4) Are the methods sensitive to the depreciation choice that the firm has used in producing its financial statements?

Fifth, the IRR estimation models are largely based on the idea of regular development uninterrupted by structural changes or other major one-time events causing exceptional capital investment peaks. Our fifth question relates to this aspect:

5) Are the methods sensitive to capital investment shocks?

An economic time series is made up by several constituents. These are the growth trend, the business cycle, the seasonal variation and the noise. Furthermore, there can be regular or irregular shocks. The growth trend and the business cycle are relevant in this paper. Seasonal variations are intra-year. Thus they do not arise in our research questions. It is true that the economic activities of the firm are continuous in nature. However, the financial data used for the profitability estimation in the methods under observation use discontinuous observations from the annual statements.

Our enquiry does not include a random variation in the firm's time series. Not including the noise component enables an unbiased evaluation of the best performance of the IRR estimation. Furthermore, our contingency numerical experiments indicated that a noise component would not affect the nature of our results. It is not needed in this paper even if it could easily be included into the data generation process to be presented.

We tackle the presented questions as follows. If the true profitability of the firm under observation were known then the various methods could be evaluated and compared. However, this is not the case for real-life business enterprises. Therefore, we construct a realistic simulation procedure that generates the central financial time series of the firm. Using simulated data will allow us to know the true IRR in advance. This enables evaluating and comparing the IRR estimation methods under observation. For a view on the simulation approach in accounting see e.g. Henderson, Peirson and Brown (1992; 32–34).

An alternative approach would be to tackle the comparison based on samples of actual business firms. For example Salamon (1985) studied the empirical estimates of the IRR and ARR for a five year sample of 197 Compustat firms. Butler, Holland and Tippet (1994) considered the relationship in terms of a stochastic process. Also see O'Hanlon (1995) for a recent review on (stochastic) ARIMA modelling of the firm's earnings variables including the accounting rate of return time series. Compared to the statistical approaches requiring samples of actual company financial statements our simulation approach has the advantage of having the true IRR available as the exact benchmark.

# 2. THE MODELS

## 2.1 The Simulation Engine

The simulation engine, the data generation and the simulation design are presented below. For a detailed presentation see Salmi and Virtanen (1995a; 7–22).

The capital investments are generated by the following process with a trend, cyclical and a potential shock component

(1) $g_t = g_0 (1 + k)^t \{1 + A \sin[(2\pi t/C) + \phi]\} [1 + \delta_{t\tau} S]$,

where $\delta$ is Kronecker's delta, i.e.

(2) $\delta_{t\tau} = 1$ when $t = \tau$, and 0 otherwise.

In the above we have denoted

$g_t$ = capital expenditures in year t
$k$ = growth rate trend of the capital investments
$A$ = amplitude of the business cycle
$C$ = length of the business cycle
$\phi$ = phase adjustment for the business cycle
$S$ = capital investment shock coefficient
$\tau$ = the year of the capital investment shock ($\tau = \infty$ for no shock in the simulation).

Using this capital investment generating process produces financial time series which resemble the time series profiles observed on actual business firms. See e.g. the sample of the time series drawn in Salmi et al. (1984; 46–48).

The capital investments $g_t$ induce (later) cash inflows which can be defined in terms of a contribution distribution $b_i$ where $i = t + 1,\ldots,t + N$. This formulation is based on Ruuhela (1972). The cash flow profiles in Ijiri (1979), Salamon (1982) and Gordon and Hamer (1988) represent the same idea of contributions induced by the capital investments of the firm.

The total contribution $f_t$ in year t is cumulated from the contributions from the capital investments made in the earlier years:

(3) $$f_t = \sum_{i=1}^{\min(N,t)} f_{ti} = \sum_{i=1}^{\min(N,t)} b_i \, g_{t-i}$$



where

$f_t$ = cash inflow in year t
$f_{ti}$ = absolute contribution in year t from capital investment i years back

$b_i$ = relative contribution from capital investment i years back
N = life-span of every capital investment project.

As is the custom in growth models, constant returns of scale on the capital investments are assumed. In other words, when the firm grows, there are no economics of scale. See e.g. the standard reference Levhari and Shrinivasan (1969; 153).

The true internal rate of return r for the simulated data can be readily solved e.g. using the bisection method of numerical analysis from

$$(4) \quad \sum_{i=1}^{N} b_i (1 + r)^{-i} = 1.$$

For the bisection method see any standard text-book of numerical analysis such as Conte (1965; 39–43).

In the simulations a conventional cash-flow contribution pattern will be used. It is a well-known fact that under non-conventional cash flows (more than one sign alteration) there can be multiple or no real roots for the internal rate of return r in Equation (4). See e.g. Teichroew, Robichek and Montalbano (1965). This problem does not arise in the simulations.

The accountant's profit $p_t$ is defined by the cash inflow less depreciation $d_t$

$$(5) \quad p_t = f_t - d_t.$$

The book value $v_t$ of the firm at the end of period t is defined by

$$(6) \quad v_t = v_{t-1} + g_t - d_t.$$

The simulations are conducted separately for three different depreciation methods that might be employed by the firm in its financial statements. The methods are the theoretical annuity depreciation, the business practice based straight-line depreciation and double declining balance depreciation.

The formula for the annuity depreciation is

$$(7) \quad d_t = f_t - r\, v_{t-1}.$$

As is evident, a circular reasoning is involved, since the true internal rate of return r is assumed known in the formula. The annuity depreciation method is included for theoretical reasons to verify whether the simulation and the profitability estimation algorithm give the expected results.

The straight-line depreciation method formula is

$$d_t = \sum_{i=1}^{\min(N,t)} (1/N)\, g_{t-i}. \tag{8}$$

The double declining balance depreciation method formula is

$$d_t = \sum_{i=1}^{\min(N,t)} (2/N)\,(1-2/N)^{i-1}\, g_{t-i}. \tag{9}$$

For the last year of the life-span all the remaining book value of the relevant investment is depreciated. Double declining balance depreciation is a decreasing depreciation used in the U.S. practice. See Davidson and Weil (1977).

## 2.2 Kay's IRR Estimation Model

The simulation approach developed in the above is applied in this paper to analyze and evaluate the three IRR estimation models. Before any IRR estimation method can be applied on the simulated (or actual financial) statements, the IRR estimation method must be made operational for the financial data available. Kay (1976) presented a model for deriving IRR from ARR. For Kay's model we have from Kay (1976; 451), Salmi and Luoma (1981; 25), Peasnell (1982a; 371) and Salmi and Virtanen (1995a; 16)

$$IRR = [\sum_{t=2}^{n} p_t\,(1+IRR)^{-t}]/[\sum_{t=2}^{n} v_{t-1}\,(1+IRR)^{-t}]. \tag{10}$$

where

n = length of the observation period (number of years under observation for the profitability estimation)

The internal rate of return estimate IRR is solved from the above formula by numerical iteration. For the conditions of convergence see Steele (1986; 2–4). Our iteration procedure is a Turbo Pascal 7.0 computer program documented in Salmi and Virtanen (1996).

## 2.3 Ijiri's IRR Estimation Model

Ijiri (1979) presented an IRR estimation model based on the concept of the cash recovery rate, CRR. He derived the following relationship between CRR and IRR (Ijiri 1979; 259)

$$CRR = IRR/[1-(1+IRR)^{-N}]. \tag{11}$$



When the CRR is known, the corresponding value of IRR can be solved by numerical iteration from Formula (11). The iteration procedure is documented in Salmi and Virtanen (1996).

The cash recovery rate CRR can be defined as the ratio between the cash inflows from capital investments and the outstanding gross capital investments. Ijiri (1980; 55) presents the calculation of the CRR from published financial statements as

(12) CRR = Cash Recoveries / Gross Assets

where

Cash Recoveries = (Funds from Operations)
+ (Proceeds from Disposal of Long-Term Assets)
+ (Decrease in Total Current Assets)
+ (Interest Expense)

and

Gross Assets = (Total Assets) + (Accumulated Depreciation),
averaged between beginning and ending balances.

In our simulation model the cash recoveries are simply equivalent to $f_t$. The gross assets must be discussed in more detail. The total assets are given directly by the book value $v_{t-1}$. First, when the total assets have been defined the accumulated depreciation must be assessed to get the gross assets. Second, the beginning instead of the average book values are used.

In financial statement analysis practice the accumulated depreciation is typically obtained by cancelling backwards the depreciations for a suitable span of years. In analysis practice the choice of the backwards span tends to be somewhat arbitrary. However, it is possible to show that given the average life-span of the capital investments and a constant level annual depreciations, the accumulated depreciation will be given by accumulating the depreciations from half the average life-span. This choice will be used as the best approximation for all the depreciation profiles.

Instead of using averaging between the annual beginning and ending book values we use the beginning values $v_{t-1}$. This leads to more accurate results when discrete instead of continuous approach is used. This is in line with the treatment of Kay's model in Salmi and Luoma (1981), Peasnell (1982a) and Salmi and Virtanen (1995a; 16).

The estimates of the annual cash recovery rates $CRR_t$ are calculated from

(13) $CRR_t = f_t/V_{t-1}$

264 where $V_t$ denotes the gross assets at the end of year t calculated from

(14) $V_t = v_t + \sum_{i=0}^{N/2-1} d_{t-i}$.

The calculated $CRR_t$ values are averaged and the average is substituted as CRR into Formula (11) in line with Ijiri (1980). Ijiri's IRR estimate can then be iterated from Formula (11). (For mathematical simplicity, N is assumed an even integer).

Since we are using a simulation approach with a fully known engine to generate the observations, we also have the option to calculate the exact accumulated depreciation. This enables us to differentiate between the sources of the error in the IRR estimate. The components of the error are the error due to Ijiri's method and the error due to the approximation of the accumulated depreciation.

### 2.4 Average ARR Method

Much of the discussion, ever since Vatter (1966), in the ARR vs. IRR debate has centered around the question whether the ARR is a good approximation of the IRR. Instead of re-entering the deductive debate we revisit this question by using the averaged ARR in our simulations as the third IRR estimating method, and comparing its performance to Kay's and Ijiri's methods. The choice of the average ARR method is prompted by the fact that accounting practitioners routinely use and are comfortable with the concept of annual profits and return on investment. Employing averaged ARR as the IRR estimate can be considered a direct extension of this business practice.

The average ARR is calculated from

$$(15) \quad ARR = [1/(n-1)] \sum_{t=2}^{n} p_t / v_{t-1}.$$

Technically, an average can be calculated as an arithmetic average or a value-weighted average. We use the former for two reasons. First, the arithmetic average is in line with straightforward business practice. Second, an average with a large fairly stable denominator is very little affected by the choice of the averaging method. As was done in applying Kay's and Ijiri's methods the beginning book values are used in the denominator instead of the annual averages.

## 3. SIMULATION

### 3.1 Simulation Design



The controllable parameters and the logical structure of the simulation design are depicted in Figure 1. Table 1 lists the values of the parameters that produce the different simulation cases analyzed in this paper.

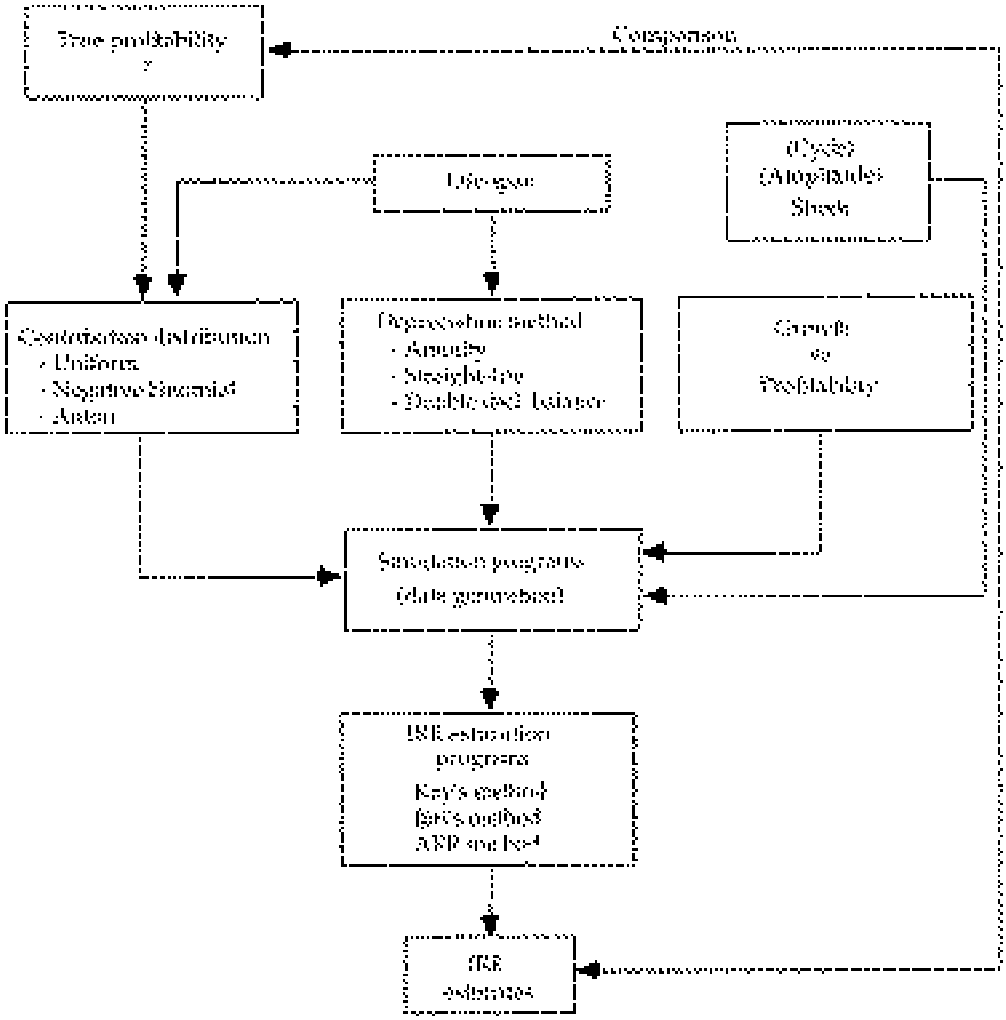

***FIGURE 1. The structure of the simulation design.***

An example of the simulated financial statement standard-disclosure time series is given in Table 2. The observation period in Table 2 is 13 years from the simulated year 22 to 34 (the lines not denoted by the *). The example is for the case of the negative binomial contribution distribution with a true profitability of 12%, a growth trend of 8%, medium amplitude (A = 0.50) of cyclical fluctuations, a life-span of 20 years of the capital investments, and a double declining balance depreciation of 10%.

Figure 2 visualizes the example financial time series.

***TABLE 1. The variation of the parameters in the simulation runs.***

| PARAMETER | SYMBOL | VALUES |
|---|---|---|
| FIRST INITIAL INVESTMENT | $g_0$ | 100.00 |
| GROWTH RATE | k | 0.08 |
| TRUE INTERNAL RATE OF RETURN | r | 0.04, 0.08, 0.12, 0.16 |
| AMPLITUDE | A | 0.00, 0.50, 1.00 |
| CYCLE LENGTH | C | 6 YEARS |
| TECHNICAL PHASE ADJUSTMENT | $\phi$ | $\pi/6$ |
| SHOCK TIMING | $\tau$ | NONE, EARLY, LATE |
| SHOCK COEFFICIENT | S | 0, 5.309, 17.924 |
| LIFE-SPAN OF INVESTMENTS | N | 16, 20, 24 |
| LENGTH OF OBSERVATION PERIOD | n | 13 (YEARS 22–34) |
| CONTRIBUTION DISTRIBUTION | | UNIFORM, NEGATIVE BINOMIAL, ANTON |
| DEPRECIATION METHOD | | ANNUITY, STRAIGHT-LINE, DECLINING |

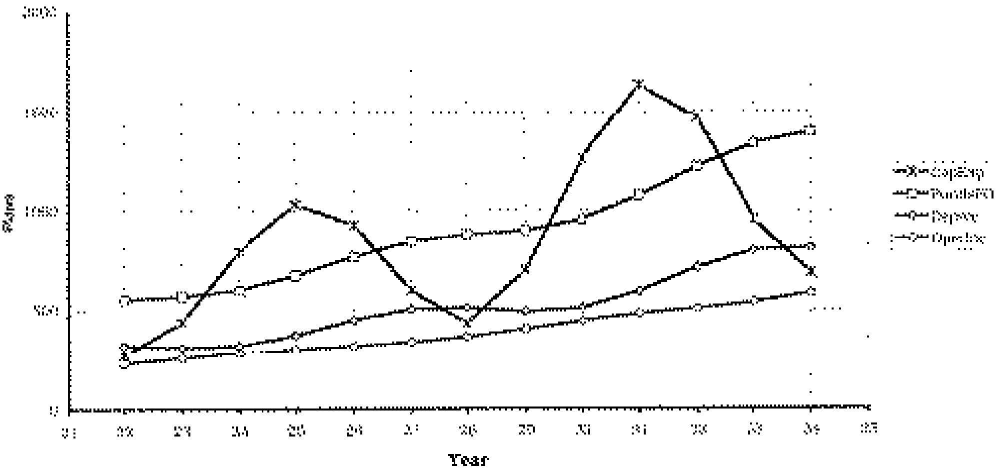


***FIGURE 2. Visualization of simulated observations: negative binomial contribution, declining balance depreciation, no shock, growth 8 %, IRR 12 %, amplitude 0.50.***

***TABLE 2. Example of simulated observations.***

| YEAR t | CAPITAL EXPENDITURE $g_t$ | FUNDS FROM OPERATIONS $f_t$ | DECLINING DEPRECIATION $d_t$ | OPERATING INCOME $p_t$ | NET BOOK VALUE $v_t$ |
|---|---|---|---|---|---|
| * 0 | 100.00 | 0.00 | 0.00 | 0.00 | 100.00 |
| * 1 | 162.00 | 10.70 | 10.00 | 0.70 | 252.00 |
| * : | : | : | : | : | : |
| * 21 | 377.53 | 668.10 | 317.10 | 351.00 | 3034.50 |
| 22 | 271.82 | 710.99 | 321.17 | 389.82 | 2985.15 |
| 23 | 440.35 | 737.24 | 310.00 | 427.24 | 3115.50 |
| 24 | 792.64 | 775.46 | 319.82 | 455.64 | 3588.33 |
| 25 | 1027.27 | 850.80 | 372.23 | 478.57 | 4243.37 |
| 26 | 924.54 | 957.45 | 448.45 | 509.00 | 4719.46 |
| 27 | 599.10 | 1060.20 | 503.20 | 557.00 | 4815.37 |
| 28 | 431.35 | 1128.25 | 509.66 | 618.59 | 4737.06 |
| 29 | 698.79 | 1169.91 | 491.93 | 677.98 | 4943.92 |
| 30 | 1257.83 | 1230.56 | 507.51 | 723.04 | 5694.23 |
| 31 | 1630.15 | 1350.11 | 590.68 | 759.43 | 6733.70 |
| 32 | 1467.13 | 1519.35 | 711.63 | 807.71 | 7489.20 |
| 33 | 950.70 | 1682.40 | 798.51 | 883.89 | 7641.38 |
| 34 | 684.50 | 1790.40 | 808.77 | 981.62 | 7517.11 |

Table 3 presents the corresponding cumulative depreciation and the gross book value needed later for applying Ijiri's IRR estimation method. As explained in Section 2.3 they are calculated for our error analysis in two different ways. The first two columns are calculated with the exact cumulative depreciation. In a simulation approach this is possible since the engine producing the financial data is known accurately. The two last columns are calculated in line with what could be done with actual data from business firms.



The three contribution distributions $b_i$ to be substituted into Formula (4) are the uniform distribution, the negative binomial distribution and the Anton distribution. The structure of the uniform distribution is straight-forward and easily applicable.

***Table 3. Cumulative depreciation and gross book values for Ijiri's method.***

| YEAR t | ACCURATE CUMUL. DEPRECIATION ACCUR $D_t$ | ACCURATE GROSS BOOK VALUE ACCUR $V_t$ | ESTIMATED CUM. DEPRECIATION $D_t$ | ESTIMATED GROSS BOOK VALUE $V_t$ |
|---|---|---|---|---|
| * 0 | 0.00 | 100.00 | .. | .. |
| * 1 | 10.00 | 262.00 | .. | .. |
| * : | : | : | : | : |
| *21 | 2387.14 | 5421.64 | .. | .. |
| 22 | 2562.51 | 5547.66 | .. | .. |
| 23 | 2778.03 | 5893.54 | .. | .. |
| 24 | 3029.83 | 6618.17 | .. | .. |
| 25 | 3291.86 | 7535.24 | .. | .. |
| 26 | 3541.96 | 8261.42 | .. | .. |
| 27 | 3788.08 | 8603.46 | .. | .. |
| 28 | 4066.38 | 8803.44 | .. | .. |
| 29 | 4408.96 | 9352.31 | .. | .. |
| 30 | 4807.96 | 10502.20 | .. | .. |
| 31 | 5223.77 | 11957.48 | 4374.68 | 11108.39 |
| 32 | 5620.64 | 13109.84 | 4765.15 | 12254.38 |
| 33 | 6011.21 | 13652.60 | 5253.66 | 12895.05 |
| 34 | 6452.84 | 13969.96 | 5742.62 | 13259.74 |

The application of the negative binomial distribution is presented in more detail in Salmi and Virtanen (1995a; Section 3.2). It is defined as

(16) $b_i = s\,(i + 1)\,p^2\,(1 - p)^i$ for $i = 1,2,\ldots,N$

where s is a scaling factor inducing the desired level of true profitability. With, for example, p = 0.15 this corresponds to a typical product life-cycle. For the definition and the properties of the negative binomial distribution see Fisz (1967; 167).



The Anton distribution presented in Anton (1956) is defined as

(17) $b_i = 1/N + (N-i+1)\,r\,/\,N$ for $i = 1,\ldots,N.$

It is a linearly declining contribution distribution with convenient theoretical properties. See Ruuhela, Salmi, Luoma and Laakkonen (1982; 332).

We use a growth rate (k) of 8%. Simulated data is generated to produce true profitability figures (r) on both sides of the growth rate. Although the growth rate has been made fairly realistic, the actual point is the relation between the profitability and growth. Either could be fixed and the other varied to achieve cases of low profitably (4%) compared to growth, equal rates (8%) and high profitabilities (12% and 16%) in relation to growth (8%). We have decided to fix the growth rate in the simulation and vary the profitability, but it could have easily been done the other way round.

The inclusion of the business cycle in Formula (1) is an extension of the simulation evaluation approach of Salmi and Luoma (1981). It is realistic to assume that the long-run average length of a business cycle is six years (C = 6). Three alternative amplitudes were originally used in our simulations. Since the results were found to be insensitive to the cycles, the amplitude is fixed at A = 0.50 in the presentation of the results.

The life-span of the capital investments affects the numerical values of the chosen contribution distribution and the annual depreciation figures. We assume a typical 20-year life-span. Furthermore, Ijiri's method requires an estimate of the life-span as part of the IRR estimation procedure. The effect of misestimating the life-span in Ijiri's method will be considered in the analysis section.

The time series are produced for three different depreciation methods to evaluate their effect on the results. The first two methods are the straight-line depreciation and double declining balance depreciation based on the common accounting practice. The third method to be used in the analysis is the theoretical annuity depreciation. The assumed 20-year life-span of the simulated capital investments means that the annual rate of depreciation in generating the simulated data is 5% in the straight-line method and 10% in the double declining balance method.

The robustness of a profitability estimation method can be tested by including capital investment shocks in the model. In business terms such a shock is usually related to a major deviation from the level of capital investment pattern. A shock of a five-fold order with relation to the normal capital investment level is used to test model robustness.

## 3.2 Simulation Results



### 3.2.1 Kay's Model

Our simulation analysis of Kay's method showed that the following factors affected the profitability estimates:

1) The disparity between the firm's growth and profitability,
2) The contribution distribution of the capital investments,
3) The depreciation method used by the firm.

The more detailed presentation of identifying these essential factors is presented in Salmi and Virtanen (1995a). Table 4 summarizes the simulation results.

***TABLE 4. Estimation of IRR with Kay's model, growth rate k = 8%, amplitude A = 0.50, no shock.***

| CONTRIBUTION DISTRIB. | | UNIFORM | | | NEG. BINOMIAL | | | ANTON | | |
|---|---|---|---|---|---|---|---|---|---|---|
| DEPRECIATION | | ANN | STR.L | DECL | ANN | STR.L | DECL | ANN | STR.L | DECL |
| TRUE r | 4% | 4.0 | 3.6 | 2.8 | 4.0 | 4.1 | 3.4 | 4.0 | 4.0 | 3.3 |
| | 8% | 8.0 | 8.0 | 8.0 | 8.0 | 8.0 | 8.0 | 8.0 | 8.0 | 8.0 |
| | 12% | 12.0 | 12.9 | 13.9 | 12.0 | 12.3 | 13.2 | 12.0 | 12.0 | 12.7 |
| | 16% | 16.0 | 18.3 | 20.3 | 16.0 | 17.1 | 18.8 | 16.0 | 16.0 | 17.5 |

The profitability estimates were found to be insensitive to the cyclical fluctuations and their amplitude. Therefore, the amplitude of the cyclical fluctuations is fixed at an intermediate level in the table. Large capital investments shocks turned out to be disruptive for the profitability estimation with Kay's method. The analysis of the effect of the shocks is documented in Salmi and Virtanen (1995a).

There are several theoretical assertions about the relationship between the internal rate of return and the accountants rate of return under the specific growth rates, depreciation methods and contribution distributions presented in earlier literature. Next we consider these assertions under the more general conditions of cyclical fluctuations utilizing the simulation results.

Solomon (1966; 115) posed that when the growth rate and the true internal rate of return are equal, the accountant's rate of return also becomes the same. Accordingly, the simulation for Kay's method should produce the following result. When the growth rate (k) and the true internal rate of return (r) are equal, the IRR estimate should become equal to the true internal rate of return for all depreciation methods and for all contribution distributions. This contention is confirmed by the results. See the row for r = 8% in Table 4.

Analytically, the accountant's rate of return and the internal rate of return are equal when the annuity method of depreciation is used (see e.g. Salmi and Luoma, 1981; 28 and Peasnell, 1982a; 364). The simulation results for Kay's method are in agreement with this contention for all the observed combinations of growth vs. profitability and for all contribution distributions. See the columns marked ”Ann” in Table 4.

It is a well-known result that the theoretical annuity depreciation method and the business practice straight-line depreciation method yield the same depreciation if the contribution distribution for the capital investments is the Anton distribution. See Solomon (1971; 168 footnote) for references. Consequently, for the Anton contribution distribution the simulations should produce the same IRR estimate for the straight-line depreciation as it does for the annuity depreciation. Also this contention is confirmed. Compare the columns marked ”Ann” and ”Str.l” below ”Anton” in Table 4.

The results discussed in the above were under very special, theoretical conditions. The rest of the entries in Table 4 show the generic effects of the growth/profitability relationship (reflecting the firms long-term growth strategies), the contribution distribution (reflecting the firm’s investment opportunities) and the depreciation method (reflecting the firm’s accounting choices).

The simulation analysis readily agrees with Kay’s results that when the firm’s growth exceeds its true profitability ($r < k$) Kay’s method under-estimates the true profitability ($r$). When $r > k$ IRR is an over-estimate of $r$. The error increases as the difference between the growth and the true profitability widens. However, the numerical results in Table 4 are affected by all the three factors, not only the growth/profitability relationship. The different factors can have compensating influence on the total IRR estimation error, as for the deviating case of $r = 4\%$, $k = 8\%$, negative binomial distribution and straight-line depreciation, where the IRR estimate is 4.1%.

In Table 4 it is seen that under capital investment opportunities that contribute in accordance with the negative binomial distribution, or the Anton distribution, the results are more accurate than under the non-declining uniform contribution distribution.

The effect of the firm’s accounting choice appears highly important to the accuracy of the IRR estimates. The error in the estimates in Table 4 is half or less when the firm applies the straight-line depreciation method instead of the double declining balance method. This observation raises interesting accounting issues about the depreciation method choice.

In the worst cases in Table 4 Kay’s IRR estimates are off in relative terms by 25 per cent from the true profitability. The significance of the error cannot be evaluated outright. The eventual significance of estimation errors in financial information would be dependent on their effect on the management’s decision making.



### 3.2.2 Ijiri’s Model

Tables 5 to 7 present the simulation results for Ijiri’s method. In addition to the more condensed information in Table 4 for Kay’s method, Tables 5 to 7 include the results for alternative estimates of the capital investments' life-span E(N). In simulation the true life-span of the capital investments is known accurately. It is 20 years in our example. But in actual financial analysis practice Ijiri's approach requires an estimate of the life-span of the firm's investments. The different life- span alternatives are included for evaluating the reliability of Ijiri's method also in this respect. In Tables 5 to 7 the IRR estimates are presented assuming a correctly estimated life-span, an underestimate 16 years, and an overestimate 24 years.

The accumulated depreciation must be estimated from the financial statements in Ijiri's method since the gross book value is needed as is seen in Formulas (13) and (14). The results are presented when the simulated firm alternatively employs two different depreciation methods, the straight-line depreciation ("Str.l") or the declining balance depreciation ("Decl"). In accounting practice, the accumulated depreciation figure usually is an approximation based on a time series of recent financial statements. However, in a simulation approach it is possible to calculate the accumulated depreciation accurately. The accurate accumulated depreciation is presented in the "Accu" column of the tables. This particular information facilitates a decomposition analysis of the error sources in the IRR estimates.

For Kay's IRR vs. ARR based method the IRR estimates were perfectly accurate under the theoretical annuity depreciation. However, the same behavior was not repeated numerically for Ijiri's method. This is not very surprising, since CRR is not part of the theoretical relationship between IRR, ARR and the annuity depreciation. The case of annuity depreciation cannot be used as a benchmark for Ijiri's method. Therefore it is not included as a column in the tables.

The first of our research questions concerns the effect of the cyclical fluctuations on the IRR profitability estimation methods. As for Kay's model our simulations for Ijiri's method indicate that the method is not sensitive to cycles. Therefore the numerical results for different cycle amplitudes need not be presented below. Furthermore, our last research question concerns the effect of investment shocks on the profitability estimates given by the various methods. Our simulations for Ijiri's method show a similar unpredictable and disruptive effect of shocks as was observed for Kay's method. See Salmi and Virtanen (1995a) for details. The numerical results involving shocks for Ijiri's method are not displayed below since their details would not produce additional information. The omitted computer runs and results can, however, be readily reproduced since the relevant computer source codes are available to the reader in Salmi and Virtanen (1996).

As discussed in the previous section there are in literature the theoretical assertions about the relationship between the internal rate of return and the accountant's rate of return under specific growth rates, the features of the depreciation methods and the properties of the contribution distributions. These assertions do not cover the relationship between the cash recovery

***TABLE 5. Estimation of IRR with Ijiri's model, uniform contribution distribution, growth rate k = 8%, amplitude A = 0.50, no shock.***

| ESTIMATED LIFE-SPAN | | 16 YEARS | | | 20 YEAR | | | S 24 YEARS | | |
|---|---|---|---|---|---|---|---|---|---|---|
| DEPRECIATION | | ACCU | STR.L | DECL | ACCU | STR.L | DECL | ACCU | STR.L | DECL |
| TRUE r | 4% | 2.0 | 2.9 | 3.6 | 4.0 | 4.2 | 4.8 | 5.2 | 4.9 | 5.3 |
| | 8% | 6.4 | 7.5 | 8.4 | 8.0 | 8.3 | 8.9 | 8.9 | 8.5 | 9.0 |
| | 12% | 10.8 | 12.0 | 13.1 | 12.0 | 12.3 | 13.1 | 12.6 | 12.2 | 12.8 |
| | 16% | 15.1 | 16.5 | 17.7 | 16.0 | 16.4 | 17.3 | 16.4 | 15.9 | 16.6 |

***TABLE 6. Estimation of IRR with Ijiri's model, negative binomial contribution distribution, growth rate k = 8%, amplitude A = 0.50, no shock.***

| ESTIMATED LIFE-SPAN | | 16 YEARS | | | 20 YEARS | | | 24 YEARS | | |
|---|---|---|---|---|---|---|---|---|---|---|
| DEPRECIATION | | ACCU | STR.L | DECL | ACCU | STR.L | DECL | ACCU | STR.L | DECL |
| TRUE r | 4% | 2.5 | 3.4 | 4.2 | 4.4 | 4.6 | 5.2 | 5.7 | 5.4 | 5.8 |
| | 8% | 6.4 | 7.4 | 8.3 | 7.9 | 8.2 | 8.8 | 9.0 | 8.6 | 9.1 |
| | 12% | 10.2 | 11.4 | 12.4 | 11.4 | 11.7 | 12.4 | 12.3 | 11.9 | 12.4 |
| | 16% | 14.0 | 15.4 | 16.6 | 14.9 | 15.3 | 16.1 | 15.7 | 15.2 | 15.9 |

***Table 7. Estimation of IRR with Ijiri's model, Anton contribution distribution, growth rate k = 8%, amplitude A = 0.50, no shock.***

| ESTIMATED LIFE-SPAN | | 16 YEARS | | | 20 YEARS | | | 24 YEARS | | |
|---|---|---|---|---|---|---|---|---|---|---|
| DEPRECIATION | | ACCU | STR.L | DECL | ACCU | STR.L | DECL | ACCU | STR.L | DECL |
| TRUE r | 4% | 2.4 | 3.3 | 4.1 | 4.4 | 4.6 | 5.2 | 5.5 | 5.2 | 5.6 |
| | 8% | 6.4 | 7.5 | 8.4 | 8.1 | 8.4 | 9.0 | 8.9 | 8.6 | 9.1 |
| | 12% | 10.0 | 11.2 | 12.2 | 11.4 | 11.7 | 12.5 | 12.0 | 11.6 | 12.2 |
| | 16% | 13.3 | 14.6 | 15.8 | 14.5 | 14.8 | 15.7 | 15.0 | 14.5 | 15.1 |



rate and the internal rate of return. This state of matters also is clearly reflected in the simulation results as a lack of similar theoretical regularities as were observed in the results for Kay. Thus the numerical results for Ijiri's method cannot be checked against similar theoretical benchmarks as was possible for Kay's method.

Ijiri's method fares on the average at least as well in the simulations as does Kay's method both if the firm applies straight-line depreciation or declining balance depreciation. The worst cases appear in the simulation results when the profitability is low compared to the growth, life-span has been overestimated and the firm has applied the declining balance depreciation. However, there is no clear pattern in the errors. There are no cases where the error would disappear. Furthermore, there is no clear pattern to the direction and the magnitude of the error. To sum up, the method fares comparatively well in practice but fares less well in the theoretical regularity.

A decomposition of the sources of the overall error reveals a more critical picture of the quality of the estimates by Ijiri's method. The total error in Ijiri's IRR estimates is made up by three components, which individually can be larger in absolute terms than the total error, but the components of the error compensate each other. Table 8 presents one example of the decomposition of the total error.

***TABLE 8. Decomposition of the estimation error in Ijiri's method. An example with negative binomial contribution distribution, declining balance depreciation, growth rate k = 8%, amplitude A = 0.50, no shock.***

| E(N) | 16 YEARS | | | | 20 YEARS | | | | 24 YEARS | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SOURCE OF ERROR | METHOD | LIFE-SPAN ESTI | CUM DEPR CALC | TOT. | METHOD | LIFE-SPAN ESTI | CUM DEPR CALC | TOT. | METHOD | LIFE-SPAN ESTI | CUM DEPR CALC | TOT. |
| r=4% | 0.4 | –1.9 | 1.7 | 0.2 | 0.4 | 0 | 0.8 | 1.2 | 0.4 | 1.3 | 0.1 | 1.8 |
| 8% | –0.1 | –1.5 | 1.9 | 0.3 | –0.1 | 0 | 0.9 | 0.8 | –0.1 | 1.1 | 0.1 | 1.1 |
| 12% | –0.6 | –1.2 | 2.2 | 0.4 | –0.6 | 0 | 1.0 | 0.4 | –0.6 | 0.9 | 0.1 | 0.4 |
| 16% | –1.1 | –0.9 | 2.6 | 0.6 | –1.1 | 0 | 1.2 | 0.1 | –1.1 | 0.8 | 0.2 | –0.1 |

The total error is made up of the following three components. If the user of Ijiri's method knew exactly the true life-span of the capital investments and were able to calculate the accumulated depreciation figures accurately, the all the error would be attributable to the method's formal derivation. This error is listed in Table 8 in the column "Method". However, the focus of interest is on deriving the estimates for observed business firms. Hence the life span of the capital investments cannot be readily known accurately. The column "Life-span esti" displays how much of the total error is due to errors in estimating the life-span. Furthermore, obtaining the accumulated depreciation from a time series of published financial statements is not trivial and involves approximations in actual accounting practice. The column "Cum Depr Calc" reflects the resultant error. The column ”Tot.” gives the total error, which is equivalent to the error in Table 6 between the estimated IRR and the true internal rate of return.

### 3.2.3 Average ARR Method

The long-standing discussion about the relevance of the average accountant's rate of return as a surrogate of the economist's theoretical profitability comes down to the question whether the average ARR is a good approximation of the firm's internal rate of return, or whether the more complicated methods are the only avenue to a proper long-term profitability estimation. The accountant's way of evaluating annual profits is dominant in business practice. Hence the soundness of extending the ARR concept to long-term profitability estimation is of paramount practical importance and interest. Tables 9 and 10 present the simulation results for using the average ARR method.

***TABLE 9. Estimation of IRR with the Average ARR method, growth rate k = 8%, amplitude A = 0.50, no shock.***

| CONTRIBUTION DISTRIB. | | UNIFORM | | | NEG. BINOMIAL | | | ANTON | | |
|---|---|---|---|---|---|---|---|---|---|---|
| DEPRECIATION | | ANN | STR.I | DECL | ANN | STR.I | DECL | ANN | STR.I | DECL |
| TRUE r | 4% | 4.0 | 3.6 | 2.9 | 4.0 | 4.1 | 3.5 | 4.0 | 4.0 | 3.3 |
| | 8% | 8.0 | 8.0 | 8.0 | 8.0 | 8.0 | 8.1 | 8.0 | 8.0 | 8.0 |
| | 12% | 12.0 | 13.0 | 13.9 | 12.0 | 12.3 | 13.2 | 12.0 | 12.0 | 12.7 |
| | 16% | 16.0 | 18.3 | 20.2 | 16.0 | 17.1 | 18.7 | 16.0 | 16.0 | 17.4 |

The results produced by the average ARR method in Table 9 are strikingly similar to the simulation results with Kay's model in Table 4. The maximum difference in the estimates is 0.1 per cent. On a closer consideration this is not an unexpected result, since Kay's method in Formula (10) can be interpreted as an iterative weighted average ARR method.

Given the close kinship between Kay's model and the average ARR method it is interesting to observe in the simulation which of the theoretical contentions hold empirically for the average ARR method.

The first theoretical contention discussed in connection with Kay's model was Solomon's position that when the growth rate and the true internal rate of return are equal, the accountant's rate of return also becomes the same. For Kay's method no numerical deviation from this equivalence could be observed. For the average ARR method the same observation is made when there are no cyclical fluctuations. However, with the cyclical fluctuations in Table 9 the relationship no longer holds accurately, but the deviation is marginal.

In Table 9 the theoretical contention holds about the equivalence of the IRR and the ARR under the theoretical annuity depreciation method. Furthermore, if the contributions from the capital investments follow the Anton distribution, the straight-line depreciation method results remain equivalent to the annuity depreciation results.

As observed, the simulated results with the average ARR method are very similar to the results with Kay's model. Therefore the behavior of the error is similar to what was already presented in connection with Kay's method.

The last of our research questions concerned the models' sensitivity to capital investment shocks. Table 10 shows that the direction and the magnitude of the additional error caused by capital investments shocks do not follow a fixed pattern. However, as would be intuitively expected, when the simulated firm uses the declining balance depreciation, the existence of the shock seems to lower the IRR estimate. Under straight-line depreciation the timing of the shock seems to dominate. Thus, for an early shock and low profitability there is a marked increase in the IRR estimate.

***TABLE 10. Estimation of IRR with the Average ARR method, negative binomial contribution distribution, growth rate k = 8%, amplitude A = 0.50, realistic shock S = 5.309.***

| SHOCK | | NONE | | | EARLY ($\tau = 24$) | | | LATE ($\tau = 30$) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| DEPRECIATION | | ANN | STR.l | DECL | ANN | STR.l | DECL | ANN | STR.l | DECL |
| TRUE R | 4% | 4.0 | 4.1 | 3.5 | 4.0 | 4.4 | 3.5 | 4.0 | 4.1 | 3.0 |
| | 8% | 8.0 | 8.0 | 8.1 | 8.0 | 8.2 | 8.0 | 8.0 | 7.8 | 7.3 |
| | 12% | 12.0 | 12.3 | 13.2 | 12.0 | 12.4 | 13.1 | 12.0 | 12.0 | 12.2 |
| | 16% | 16.0 | 17.1 | 18.7 | 16.0 | 17.1 | 18.6 | 16.0 | 16.6 | 17.5 |

## 3.3 Comparison of Results from the Different Methods

In comparing the different methods the following aspects are relevant: numerical performance, theoretical foundations and practical applicability.

First, consider numerical performance. In our simulations the relevant parameters were given values that should put them in a realistic range with regard to actual business firms. Within the observed range none of the methods clearly outperforms the others in the simulation. The deviations in Kay's and the average ARR method were more regular and predictable than the deviations in Ijiri's method. The number of potential sources of errors in Ijiri's method was greater. Since the errors in Ijiri's method partly compensated for each other the resulting total error, while less predictable, was no worse than for the other methods.

In the simulations of the present paper each of the boxes in the different tables can be considered "equally weighted". One potential direction of further research would be to adopt a numerical index to compare the numerical performance of the methods with each other. For this purpose it would be necessary to estimate from factual business observations the relative frequencies of the different combinations of the key parameters.

Second, consider theoretical foundations. In the light of the simulation results Kay's method came out as the theoretically best founded, with the average ARR method very close by. The ARR equality to IRR when the growth rate and the IRR agree, the theoretical annuity depreciation method's IRR-conformance, and the posed relationship of the annuity and straight-line depreciation methods under Anton contribution distribution all were confirmed in the simulations with Kay's method. Ijiri's method did not conform to any of these expected theoretical propositions. This fact casts serious doubts of the theoretical validity of the method despite its relative reliability in the numerical simulation. The conclusion is that Ijiri's method can be regarded as an elaborate, good rule of thumb.

Last, consider practical applicability. In this area the average ARR method has the outstanding merit of being directly based on established accounting practice of performance measurement. It would be trivial to use computers to calculate Kay's IRR estimates in business practice. However, in our opinion, the marginal improvement compared to the average ARR method does not compensate the obvious disadvantages of having to "sell" this method to the users of financial information. In this light, it is our recommendation to choose for long-term profitability estimation the average ARR method over both Kay's and Ijiri's methods.

## 4. SUMMARY AND CONCLUSIONS

This paper used a realistic simulation approach developed by the authors to evaluate three internal rate of return long-term profitability estimation models. Five research questions were posed concerning Kay's, Ijiri's and the average ARR methods.

First, our approach included capital investment cycles into the simulation analysis. It was observed that none of the three methods is sensitive to cyclical fluctuations. This is an important result because it confirms the applicability of the methods beyond the usual steady state assumptions.



Second, the true pattern of contributions from the firm's capital investments is not known. It was observed that the three methods can be sensitive to the contribution distribution. The existence and the magnitude of the effect of the shape of the contribution distribution on the

IRR estimates is dependent interactively on the depreciation methods applied by the firm and the relationship between growth and profitability. The conclusion is that contribution distribution of the firm's capital investments can have an effect of the quality of the IRR estimates given by the three IRR estimation methods. Furthermore, contrary to the other two IRR estimation methods Ijiri's method requires an estimate of the life-span of the firm's capital investments. The reliability of the IRR estimates by Ijiri's method are highly sensitive to the quality of the life-span estimate.

Third, it was observed that the reliability of the IRR estimates of all the three methods is highly sensitive to the relationship between the underlying true profitability and the firm's growth rate. In accordance to our simulation results the discrepancy between the true growth and profitability is the dominating source of the error in the IRR estimates in all the three methods. Furthermore, the other sources of errors in the IRR estimates interact with and can be aggravated by the growth-profitability discrepancy. This indicates that for better IRR estimation methods the growth-profitability discrepancy should be an integral part of such a method.

Fourth, the depreciation method applied by the firm in its financial statements can affect the IRR estimation result in concert with the contribution distribution of the capital investments. Also this effect is strongly related to the growth-profitability discrepancy. For example, a worst case of the interactive effect appears in our simulation for Kay's and the average ARR method for a fast growing firm with low profitability using declining balance depreciation method in a situation where the contribution from the capital investments follows the uniform distribution.

Fifth, our simulations indicate an unpredictable and disruptive effect of capital investment shocks of the both the validity and reliability of the IRR estimation methods. The effect of the capital investments shocks is to cause outliers in the financial data and hence disrupt the long-term profitability estimation.

To conclude, considering the various facets discussed in this paper, the accounting practice based average ARR method seems to be a good choice for the long-term profitability estimation. However, for fast growing firms with low profitability and for slow-growth firms with good profitability the long-term profitability estimates should be interpreted with much caution. On the other hand, the average ARR method can be safely used when a firm has comparable growth and profitability even when there are ordinary fluctuations in the capital investment intensity. ■

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