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A Fractional Integration Analysis of the Swedish Economy, 1861–1988

ABSTRACT

In this article we examine the stochastic behaviour of eleven long Swedish macroeconomic time series by means of using fractionally integrated techniques. Using a version of the tests of Robinson (1994) that permits us to test unit and fractional roots in raw time series, the results show that the order of integration of the original series is higher than 1 in practically all cases. However, using the log-transformed data, the unit root null cannot be rejected, implying that the growth rate series are I(0) and do not possess long memory.

Keywords: *Long memory; Fractional integration; Macroeconomic time series*

JEL Classification: *C22.*

1. INTRODUCTION

In this article we analyse eleven long Swedish macroeconomic time series, annually, from 1861 to 1988 by means of using fractionally integrated techniques. Some of these series were examined by Englund, Persson and Svensson (EPS, 1992), studying their business cycle characteristics. They filtered the series to achieve stationarity and then, estimated the spectral density and applied a band-pass filter due to Priestley (1981) to remove all variation at the frequencies other than those of interest. The main conclusion of EPS (1992) was that the Swedish business cycle (with length of 3 to 8 years) was uniform across different epochs of the Swedish eco-

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nomic history. In another recent paper, Skalin and Terasvirta (1999) take another look at the Swedish economy and test for linearity in the series analysed by EPS (1992), concluding that non-linear models may be more appropriate to analyse these series. In both of these papers, they assume that the series have unit roots, i.e., they take first differences on the original data in order to get I(0) stationary series.

Long macroeconomic time series have been rather extensively analysed in the econometric literature. For many years, the key question was to investigate if such long series have a unit root as opposed to the alternative that the series are stationary around a linear time trend (see, Nelson and Plosser, 1982 and subsequent work). However, in the last few years, a new approach of modelling macroeconomic time series in terms of fractionally integrated models has emerged. The unit root models appear then merely as a particular case of long memory processes. In this article we take this latter approach and model eleven Swedish macroeconomic series in terms of I(d) statistical models. We use a version of the tests of Robinson (1994) which permits us to test unit and fractional roots in raw time series. These tests are briefly described in Section 2. In Section 3, they are applied to the Swedish economy while Section 4 contains some concluding comments.

2. TESTING OF I(D) HYPOTHESES

For the purpose of the present paper, we define an I(0) process $\{u_t, t = 0, \pm 1, ...\}$ as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. In this context, we say that a given raw time series $\{x_t, t = 0, \pm 1, ...\}$ is I(d) if

$$(1 - L)^d \, x_t \; = \; u_t, \qquad t \; = \; 1, 2, \ldots, \tag{1}$$

$$x_t \; = \; 0, \qquad t \; \leq \; 0, \tag{2}$$

where u_t is I(0) and where L means the lag operator ($Lx_t = x_{t-1}$). Note that the polynomial above can be expressed in terms of its Binomial expansion, such that for all real d,

$$(1 \; - \; L)^d \; = \; \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j \; = \; 1 \; - \; d\,L \; + \; \frac{d\,(d-1)}{2} L^2 \; - \; \ldots.$$

The macroeconomic literature has stressed the cases of d = 0 and 1, however, d can be any real number. Clearly, if d = 0 in (1), $x_t = u_t$, and a 'weakly autocorrelated' x_t is allowed for. However, if d > 0, xt is said to be a long memory process, also called 'strongly autocorrelated', so-named because of the strong association between observations widely separated in time and as d increases beyond 0.5 and through 1, x_t can be viewed as becoming ″more nonstationary″, in the sense, for example, that the variance of partial sums increases in magnitude.[1] These processes were initially introduced by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981), (though earlier work by Adenstedt, 1974, and Taqqu, 1975 shows an awareness of its representation), and were theoretically justified in terms of aggregation of ARMA processes with randomly varying coefficients by Robinson (1978), Granger (1980). Similarly, Croczek-Georges and Mandelbrot (1995), Taqqu et. al. (1997), Chambers (1998) and Lippi and Zaffaroni (1999) also use aggregation to motivate long memory processes, while Parke (1999) uses a closely related discrete time error duration model. Empirical applications based on fractional models like (1) are among others Diebold and Rudebusch (1989), Baillie and Bollerslev (1994), Gil-Alana and Robinson (1997) and Gil-Alana (2000a).

To determine the appropriate degree of integration in a given raw time series is important from both economic and statistical viewpoints. If d = 0, the series is covariance stationary and possesses 'short memory', with the autocorrelations decaying fairly rapid. If d belongs to the interval (0, 0.5), x_t is still covariance stationary, however, the autocorrelations take much longer time to disappear than in the previous case. If $d \in [0.5, 1)$, the series is not longer covariance stationary, but it is still mean reverting, with the effect of the shocks dying away in the long run. Finally, if $d \geq 1$, x_t is nonstationary and non-mean reverting. Thus, the fractional differencing parameter d plays a crucial role in describing the persistence in the time series behaviour: higher d is, higher will be the association between the observations.

Following Bhargava (1986), Schmidt and Phillips (1992) and others on parameterization on unit root models, Robinson (1994) considers the regression model,

$$y_t = \beta' z_t + x_t, \qquad (3)$$

where y_t is the time series we observe; β is a (kx1) vector of unknown parameters; z_t is a (kx1) vector of deterministic regressors, that may include, for example, an intercept ($z_t = 1$) or an intercept and a linear time trend ($z_t = (1,t)'$), and the regression errors x_t are of form as in (1). He proposes a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o : d = d_o, \qquad (4)$$

in (1) - (3) for any real value d_o. Specifically, the test statistic is given by:

$$\hat{r} = \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \qquad (5)$$

where T is the sample size and

1 Models with d ranging between –0.5 and 0 are short memory and have been addressed as anti-persistent by Mandelbrot (1977), because the spectral density function is dominated by high frequency components.

$$\hat{a} = \frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j); \qquad \psi(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 - \sum_{j=1}^{T-1}\psi(\lambda_j)\,\hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\,\hat{\varepsilon}(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right)$$

$$\hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}); \quad \lambda_j = \frac{2\pi\, j}{T}; \quad \hat{\tau} = \arg min_{\tau}\left[\sigma^2(\tau)\right]; \quad \sigma^2(\tau) = \frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j;\tau)^{-1}\, I(\lambda_j);$$

where the minimization is carried out over a suitable subset of R^m. $I(\lambda_j)$ is the periodogram of u_t evaluated under the null, i.e.,

$$I(\lambda_j) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T}\hat{u}_t\, e^{i\lambda_j t}\right|^2.$$

where

$$\hat{u}_t = (1-L)^{d_o} y_t - \hat{\beta}' w_t; \quad \hat{\beta} = \left(\sum_{t=1}^{T} w_t\, w_t'\right)^{-1}\sum_{t=1}^{T} w_t\,(1-L)^{d_o} y_t; \qquad w_t = (1-L)^{d_o} z_t,$$

and the function g above is a known function coming from the spectral density function of u_t,

$$f(\lambda;\sigma^2;\tau) = \frac{\sigma^2}{2\pi} g(\lambda;\tau), \qquad -\pi < \lambda \le \pi.$$

Note that these tests are purely parametric and therefore, they require specific modelling assumptions to be made regarding the short memory specification of u_t. Thus, if ut is white noise, $g \equiv 1$, and if u_t is an AR process of form $\phi(L)u_t = \varepsilon_t$, $g = |\phi(e^{i\lambda})^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are function of τ.

Based on the null hypothesis H_o (4), Robinson (1994) established that under regularity conditions:

$$\hat{r} \rightarrow_d N(0,1) \quad as \quad T \rightarrow \infty, \tag{6}$$

Thus, we are in a classical large sample-testing situation by reasons described in Robinson (1994), who also showed that the tests are efficient in the Pitman sense, i.e., that against local alternatives of form: H_a: $d = d_o + \delta T^{-1/2}$, with $\delta \neq 0$, the limit distribution is normal with variance 1 and mean which cannot (when u_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic. An approximate one-sided 100α% level test of H_o (4) against the alternative: H_a: $d > d_o$ ($d < d_o$) will be given by the rule:

$$\text{"Reject } H_o(4) \;\; if \;\; \hat{r} > z_\alpha \;\; (\hat{r} < -z_\alpha)\text{"},$$

where the probability that a standard normal variate exceeds z_α is α. There also exist finite-sample critical values of this version of the tests of Robinson (1994), (Gil-Alana, 2000b). However, given the large sample size of the variables used in the empirical application below, we have decided to use the asymptotic critical values given by the normal distribution. This version of the tests of Robinson (1994) has been used in other empirical works in Gil-Alana and Robinson (1997) and Gil-Alana (2000a) and, other versions of his tests, based on seasonal (quarterly and monthly) and cyclical data can be respectively found in Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001a).

3. TESTING THE ORDER OF INTEGRATION IN THE SWEDISH ECONOMY

The time series data analysed in this section correspond to the following variables: GDP deflator (PY); GDP (Y); private consumption (C); manufacturing prices (PMAN); wage rates (W); number of workers (N); investments (I); exports (X); imports (M); manufacturing production (YMAN); and real wages (WPMAN), annually, for the time period 1861–1988, obtained from Skalin and Terasvirta (1999), (JAE, Vol. 14.1, database).

Denoting any of the series y_t, we employ throughout the model in (1) - (3) with $z_t = (1,t)'$, $t \geq 1$, $z_t = (0,0)'$ otherwise. Thus, under the null hypothesis H_o (4):

$$y_t = \alpha + \beta t + x_t, \qquad t = 1, 2, \ldots \tag{7}$$

$$(1 - L)^{d_o} x_t = u_t, \qquad t = 1, 2, \ldots,. \tag{8}$$

and we treat separately the cases $\alpha = \beta = 0$ a priori, unknown and $\beta = 0$ a priori, and α and β unknown, i.e., we consider the cases of no regressors in the undifferenced regression, an intercept and an intercept and a linear time trend. Also, we model the I(0) disturbances u_t to be both white noise and weakly autocorrelated.

We start with the assumption that u_t in (8) is white noise. Thus, for example, when d = 1, the differences $(1 - L)y_t$ behave, for t > 1, like a random walk when $\beta = 0$, and a random walk with a drift when $\beta \neq 0$. However, we report across this section the test statistics not merely for the case of d = 1 but for d = 0, (0.25), 2, thus including also a test for stationarity (d = 0.5); for I(2) processes (d = 2), as well as other fractionally integrated possibilities.[2]

The test statistic reported across Table 1 (and also in Tables 2, 3 and 5–7) is the one-sided one corresponding to (5), so that significantly positive values of this are consistent with orders of integration higher than d_o, whereas significantly negative ones are consistent with alternatives of form: $d < d_o$. A notable feature observed in Table 1, in which u_t is taken to be white

2 Negative values of d were also considered and the null hypothesis was then rejected in all cases.

TABLE 1

Testing the order of integration in the Swedish economy with the tests of Robinson (1994) and white nosie u_t

Series	z_t	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
PY	---	22.78	21.91	21.61	21.00	20.34	16.05	7.08	**0.001***	-2.84
	1	22.78	21.82	20.55	19.84	19.64	16.06	7.86	**0.50***	-2.67
	(1, t)'	23.91	23.65	22.81	21.57	19.93	15.91	7.84	**0.49***	-2.68
W	---	21.83	21.13	20.98	20.73	20.26	16.95	8.23	**-0.04***	-3.23
	1	21.83	21.08	20.93	20.24	20.19	16.88	8.16	**-0.10***	-3.27
	(1, t)'	25.79	24.02	23.26	22.07	20.52	16.59	8.01	**-0.14***	-3.28
PMAN	---	21.86	20.90	20.32	19.34	14.78	6.42	**-0.06***	-2.91	-4.05
	1	19.86	18.58	17.83	17.82	14.37	7.01	**0.62***	-2.41	-3.73
	(1, t)'	23.16	22.97	21.56	19.05	14.55	7.23	**0.72***	-2.41	-3.73
YMAN	---	25.82	25.12	24.36	19.15	8.05	**0.92***	**-1.54***	-2.57	-3.16
	1	24.82	23.61	22.13	18.69	8.00	**0.89***	**-1.55***	-2.58	-3.17
	(1, t)'	29.10	28.00	24.98	18.21	8.00	**1.23***	**-1.51***	-2.58	-3.17
Y	---	25.45	25.02	24.96	22.84	11.23	**0.44***	-2.76	-3.75	-4.28
	1	24.45	23.10	21.92	21.40	11.10	**0.21***	-2.93	-3.85	-4.32
	(1, t)'	29.25	28.72	26.90	22.22	11.27	**0.96***	-2.86	-3.86	-4.32
X	---	23.57	22.86	22.52	19.19	9.62	**0.73***	-2.55	-3.61	-4.13
	1	22.57	21.49	20.48	18.60	9.51	**0.69***	-2.56	-3.61	-4.13
	(1, t)'	27.12	26.84	24.69	19.40	9.63	**1.04***	-2.49	-3.61	-4.13
M	---	23.34	23.32	21.58	14.66	4.08	**-1.62***	-3.57	-4.34	-4.74
	1	23.34	21.92	19.64	14.17	3.97	**-1.61***	-3.59	-4.35	-4.74
	(1, t)'	27.79	26.77	22.81	14.22	4.09	**-1.52***	-3.58	-4.35	-4.74
WPMAN	---	25.30	24.98	24.56	19.14	7.85	**0.66***	-2.00	-3.14	-3.78
	1	25.30	23.36	22.26	18.69	7.86	**0.60***	-2.08	-3.02	-3.81
	(1, t)'	28.30	27.35	24.47	17.94	7.88	**0.85***	-2.08	-3.20	-3.82
C	---	25.74	25.22	24.72	19.09	7.36	**-0.47***	-2.20	-3.54	-4.32
	1	25.74	23.16	21.20	17.94	7.24	**0.42***	-2.06	-3.29	-4.04
	(1, t)'	29.23	28.01	24.72	17.48	7.36	**0.80***	-2.01	-3.29	-4.04
I	---	25.07	25.13	23.38	16.28	5.63	**-0.26***	-2.61	-3.72	-4.32
	1	25.07	23.60	21.20	15.75	5.60	**-0.34***	-2.65	-3.74	-4.32
	(1, t)'	28.99	27.48	23.54	15.37	5.64	**-0.16***	-2.66	-3.75	-4.33
N	---	27.12	26.81	18.00	7.46	1.97	**-0.98***	-2.51	-3.40	-3.99
	1	27.12	25.12	20.54	10.22	2.99	**-0.55***	-2.25	-3.17	-3.76
	(1, t)'	19.21	18.53	15.24	9.01	2.52	**-0.57***	-2.26	-3.17	-3.76

* and in bold: Non-rejection values of the null hypothesis at the 95% significant level.

noise is the fact that the test statistic monotonically decreases with d_o. This is something to be expected in view of the fact that it is a one-sided statistic. Thus, for example, if H_o (4) is rejected with $d_o = 1$ against alternatives of form: H_a: $d > 1$, an even more significant result in this direction should be expected when $d_o = 0.75$ or 0.50 are tested. We see that the unit root null hypothesis (i.e., $d_o = 1$) is rejected for all series in favour of higher orders of integration. The most nonstationary series appear to be GDP deflator and wage rates where the null hypothesis cannot be rejected when $d_o = 1.75$. It is followed by manufacturing prices (with $d_o = 1.50$) and manufacturing production (with $d_o = 1.25$ and 1.50). For the remaining series, H_o (4) cannot be rejected when $d_o = 1.25$, implying thus, nonstationarity and non-mean-reversion behaviour. Another notable feature observed in this table is the fact that the non-rejection values coincide for the three cases of no regressors, an intercept and an intercept and a linear time trend, suggesting that the results are robust to the different specifications for the deterministic trends and implying perhaps that they are not required when modelling these series.

However, the significance of the above results may be in large part due to the un-accounted for I(0) autocorrelation in u_t. Thus, we also performed the tests imposing AR(1) and AR(2) disturbances. However, the results for most of the series in this context showed a lack of monotonicity in the value of the test statistic with respect to d_o. This lack of monotonicity may be an indication of model misspecification (as is argued for example in Gil-Alana and Robinson, 1997), but it may also be due to the fact that the AR coefficients are Yule-Walker estimates and thus, though they are smaller than one in absolute value, they can be arbitrarily close to 1. A problem then may occur in that they may be capturing the order of integration by means, for example, of a coefficient of 0.99 in case of using AR(1) disturbances. In order to solve this problem, we used another type of I(0) disturbances which are due to Bloomfield (1973) and which accommodate fairly well to the present version of the tests. Using this method, the disturbances are exclusively specified in terms of the spectral density function, which is given by

$$f(\lambda;\tau) \;=\; \frac{\sigma^2}{2\pi}\,\exp\left(2\sum_{r=1}^{m}\tau_r\cos(\lambda r)\right). \tag{9}$$

Bloomfield (1973) showed that the logarithm of the spectral density function of an ARMA(p, q) process is a fairly well-behaved function and can be approximated by a truncated Fourier series. He showed that (9) approximates the spectrum of an ARMA process well where p and q are of small values, which usually happens in economics. Like the stationary AR(p) case, this model has exponentially decaying autocorrelations and thus, using this specification, we do not need to rely on so many parameters as in the ARMA processes, which always results tedious in terms of estimation, testing and model specification. Furthermore, using this specification, u_t is stationary for all real values of τ, unlike what happens with AR processes. Formulae for Newton-type iteration for estimating the τ_r are very simple (involving no matrix inversion), updating formulae when m is increased are also simple, and we can replace $\hat{A}$ below (5) by the population quantity

TABLE 2

Testing the order of integration in the Swedish economy with the tests of Robinson and Bloomfield (1) u_t

Series	z_t	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
PY	---	11.74	11.11	10.43	10.05	8.76	6.14	2.95	**-0.33***	-2.65
	1	10.74	10.14	9.66	9.54	9.45	7.81	4.36	**0.70***	-1.89
	(1, t)'	12.65	12.10	11.25	10.36	9.22	7.20	4.17	**0.67***	-1.91
W	---	10.60	10.20	9.92	9.88	9.71	8.16	5.38	**1.45***	-1.97
	1	10.60	10.23	9.96	9.66	9.66	8.22	5.38	**1.56***	-1.90
	(1, t)'	12.89	12.44	11.50	10.54	9.55	7.58	5.05	**1.46***	-1.91
PMAN	---	10.03	9.40	9.36	8.09	5.79	3.10	**0.05***	-1.95	-3.06
	1	8.83	7.94	7.74	7.07	5.94	3.16	**0.19***	-1.94	-2.88
	(1, t)'	11.69	11.67	9.93	8.41	6.05	3.02	**0.17***	-1.92	-2.87
YMAN	---	12.75	12.68	10.62	5.63	**0.45***	-2.71	-3.94	-4.52	-4.81
	1	12.75	11.43	10.13	6.09	**0.54***	-2.65	-3.96	-4.48	-4.82
	(1, t)'	16.05	13.08	9.47	4.86	**0.50***	-2.51	-3.90	-4.48	-4.82
Y	---	13.21	12.99	12.95	10.70	5.58	**-0.21***	-3.04	-4.01	-2.65
	1	12.21	11.54	10..91	10.53	5.94	**0.05***	-3.13	-.4.15	-4.48
	(1, t)'	17.57	15.82	13.50	10.69	6.00	**0.59***	-2.96	-4.11	-4.51
X	---	11.03	10.93	10.21	7.88	3.80	**-0.50***	-3.07	-4.14	-4.55
	1	10.63	9.79	9.33	7.80	3.92	**-0.40***	-3.08	-4.14	-4.55
	(1, t)'	14.64	13.57	11.13	7.86	3.82	**-0.19***	-3.03	-4.14	-4.55
M	---	11.32	10.68	9.27	6.25	2.44	**-0.77***	-2.57	-3.42	-4.02
	1	11.32	10.10	8.62	6.09	2.29	**-0.90***	-2.57	-3.48	-4.06
	(1, t)'	14.71	12.52	9.71	6.23	2.45	**-0.69***	-2.64	-3.50	-4.05
WPMAN	---	13.08	12.19	10.20	6.32	1.89	**-1.53***	-3.09	-3.78	-4.20
	1	12.08	11.14	10.37	6.69	1.95	**-1.38***	-3.05	-3.78	-4.19
	(1, t)'	14.49	12.67	9.64	5.81	1.83	**-1.18***	-3.00	-3.80	-4.23
C	---	13.50	12.81	11.57	7.69	2.70	**-0.65***	-2.06	-2.67	-3.12
	1	12.50	11.09	9.66	7.35	2.06	**-1.35***	-2.56	-3.17	-3.54
	(1, t)'	16.72	14.48	11.51	6.74	2.19	**-1.09***	-2.58	-3.17	-3.54
I	---	12.69	11.92	9.53	5.56	**1.37***	**-1.06***	-2.29	-3.01	-3.69
	1	12.69	11.30	9.45	5.70	**1.38***	**-1.05***	-2.29	-2.99	-3.67
	(1, t)'	14.77	11.83	8.85	5.02	**1.39***	**-1.04***	-2.32	-3.07	-3.74
N	---	14.62	12.10	6.44	**1.26***	**-1.03***	-2.55	-3.42	-3.84	-4.12
	1	14.62	12.40	8.86	2.34	**-0.75***	-2.57	-3.51	-3.98	-4.30
	(1, t)'	7.93	6.57	4.51	**1.64***	**-0.80***	-2.61	-3.53	-3.97	-4.29

* and in bold: Non-rejection values of the null hypothesis at the 95% significant level.

$$\sum_{l=m+1}^{\infty} l^{-2} = \frac{\pi^2}{6} - \sum_{l=1}^{m} l^{-2},$$

which indeed is constant with respect to the τ_j (unlike what happens in the AR case). The Bloomfield model for I(0) processes, confounded with the fractional model (1) has not been very much used in previous econometric applications, (though the Bloomfield model itself is a

TABLE 3										
Testing the order of integration in the Swedish economy with the tests of Robinson and Bloomfield (2) u_t										
Series	z_t	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
PY	---	6.86	6.70	5.16	5.06	4.11	3.97	3.11	**0.93***	**-0.22***
	1	8.86	7.29	6.40	4.35	4.05	3.59	2.59	**-1.26***	**-1.59***
	(1, t)’	8.78	7.32	6.13	5.93	3.68	2.97	2.69	**1.21***	-1.69
W	---	8.63	7.29	6.58	5.80	4.10	3.29	2.40	**1.44***	**1.32***
	1	8.63	7.16	6.92	6.04	5.48	4.39	3.13	**1.23***	**-0.14***
	(1, t)’	8.96	7.56	6.15	5.95	4.76	3.44	2.07	**1.04***	**-1.10***
PMAN	---	6.49	5.06	4.80	3.55	2.78	2.65	**1.55***	**0.04***	-1.81
	1	5.49	4.46	3.60	2.52	2.32	2.28	**-0.18***	**-0.57***	-2.16
	(1, t)’	5.68	5.60	4.66	4.50	3.64	2.54	**-0.16***	**-0.54***	-2.16
YMAN	---	9.22	8.70	7.18	6.63	4.64	**1.01***	**-0.48***	-2.06	-2.80
	1	9.22	8.28	7.64	5.12	4.45	**0.93***	**-0.53***	-2.07	-2.78
	(1, t)’	12.21	8.21	7.32	5.26	4.74	2.01	**-0.22***	-2.08	-2.79
Y	---	9.17	8.55	7.91	6.29	3.69	2.95	**-0.07***	-2.50	-3.06
	1	9.17	8.66	7.93	7.54	6.65	2.33	**1.14***	**-0.97***	-1.94
	(1, t)’	11.53	10.23	9.61	8.37	7.11	**1.46***	**-0.35***	**-1.15***	-2.14
X	---	8.55	7.93	6.81	5.30	4.59	3.09	**0.11***	**-1.26***	-2.51
	1	7.53	6.69	6.17	5.61	4.40	2.96	**0.03***	**-1.27***	-2.47
	(1, t)’	10.42	8.07	7.42	6.73	4.62	4.03	**0.57***	**-1.23***	-2.48
M	---	9.84	8.00	7.22	6.72	6.18	2.40	1.71	**-0.45***	-1.75
	1	7.84	6.61	5.68	4.66	4.13	3.15	**1.44***	**-0.65***	-1.89
	(1, t)’	12.42	10.12	8.94	5.48	4.19	1.82	**1.46***	**-0.71***	-1.83
WPMAN	---	12.39	11.30	10.58	7.11	4.94	2.90	**-1.04***	-2.48	-2.84
	1	10.39	9.67	7.25	6.60	4.83	1.98	**-0.37***	**-1.48***	-1.93
	(1, t)’	13.26	11.03	9.00	8.56	4.98	**0.85***	**0.59***	**-1.57***	-2.13
C	---	8.65	9.17	7.97	5.28	2.28	**0.17***	**-1.07***	**-1.50***	-3.63
	1	8.65	7.35	6.91	5.16	1.64	**0.24***	-1.90	-2.42	-2.59
	(1, t)’	10.72	9.28	8.65	5.23	1.81	**-1.20***	-1.74	-2.44	-2.58
I	---	9.73	8.86	8.40	7.37	3.23	3.09	**-0.06***	-1.85	-2.61
	1	9.73	8.42	7.88	5.70	4.52	2.86	**-1.14***	-1.74	-2.55
	(1, t)’	12.47	10.97	9.19	5.84	3.26	2.51	**-1.08***	**-1.43***	-2.19
N	---	8.99	7.87	4.73	**0.63***	**0.01***	-1.64	-3.01	-3.86	-3.87
	1	8.99	8.18	6.13	2.33	**1.30***	**0.30***	-1.75	-2.59	-2.80
	(1, t)’	5.20	4.09	3.84	2.82	**1.20***	**0.19***	-1.82	-2.57	-2.72

* and in bold: Non-rejection values of the null hypothesis at the 95% significant level.

well-known model in other disciplines, eg. Beran, 1993), and one by-product of this work is its emergence as a credible alternative to the fractional ARIMAs which have become conventional in parametric modelling of long memory. Among the few examples found in the literature are Gil-Alana and Robinson (1997), Velasco and Robinson (1999) and Gil-Alana (2001b).

Tables 2 and 3 show the results of the tests of Robinson (1994) based on Bloomfield (1973) disturbances with m = 1 and 2 respectively. If m = 1, GDP deflator, wage rates and manufacturing prices appear as the most nonstationary series, with d oscillating between 1.50 and 1.75; for GDP, exports, imports, real wages, consumption and investment, the null cannot be rejected when d is 1.25, and the unit root null hypothesis cannot be rejected in case of investment and number of workers. Therefore, the results are very similar to those reported in Table 1 for the case of white noise disturbances, though the null hypothesis of a unit root cannot be rejected now in some cases. If m = 2, (Table 3), the results are less conclusive though again GDP deflator and wage rates seem to be the most nonstationary series while the number of workers appears as the less nonstationary one.

In order to be a bit more precise about which might be the appropriate degree of integration of the series, we again performed the tests of Robinson (1994) for each type of regressors and each type of disturbances, but this time for a range of values of do with a grid of 0.01, and took the value of d_o which produces the lowest statistic in absolute value across d. The results are given in Table 4.

We see that the values of d are higher than 1 in all cases except for the number of workers with Bloomfield (1) disturbances. They oscillate widely depending on the series. Thus, for GDP deflator, wage rates and manufacturing prices, these values are in practically all cases higher than 1.50 while for consumption, investment and number of workers, they never exceed 1.30.

Tables 5–8 reproduces the results of Tables 1–4 but based on the log-transformed data. Surprisingly and contrary to the previous tables, the unit root null hypothesis cannot be rejected in many series. Starting with white noise u_t, (in Table 5), we see that GDP deflator is the only series where we reject the unit root null for all types of regressors, and the non-rejection values for this series are 1.25 if we do not include regressors and 1.50 and 1.75 with an intercept or with an intercept and a linear time trend. For the remaining series, we always find at least one case where the unit root null cannot be rejected. Also, higher orders of integration are plausible for wage rates, manufacturing prices and number of workers while d = 0.75 cannot be rejected in case of imports and investments. If we permit Bloomfield disturbances, the unit root model cannot be rejected for any series though smaller orders of integration are also plausible in many cases.

Table 8 corresponds to Table 4 but using the log-transformed data. We see there that the values of d are smaller than 1 in many cases, implying that mean reversion may occur in this context. Comparing the results in this table with those in Table 4 we observe smaller degrees of integration in all cases and for all series. Finally, the fact that the unit root cannot be rejected in most of the cases when the data are in logs may suggest that the growth rate series are I (0) and do not possess long memory behaviour.

TABLE 4				
Values of d which produces the lowest statistics				
Series	z_t	White noise	Bloomfield (1)	Bloomfield (2)
PY	---	1.75	1.72	1.90
	1	1.78	1.81	1.72
	(1, t)’	1.78	1.81	1.87
W	---	1.75	1.84	2.04
	1	1.75	1.86	1.92
	(1, t)’	1.74	1.85	1.86
PMAN	---	1.50	1.52	1.79
	1	1.54	1.51	1.46
	(1, t)’	1.54	1.52	1.49
YMAN	---	.132	1.03	1.37
	1	1.32	1.03	1.39
	(1, t)’	1.33	1.03	1.34
Y	---	1.27	1.24	1.49
	1	1.26	1.25	1.60
	(1, t)’	1.29	1.28	1.43
X	---	1.29	1.22	1.56
	1	1.28	1.22	1.54
	(1, t)’	1.30	1.24	1.61
M	---	1.15	1.18	1.63
	1	1.15	1.17	1.64
	(1, t)’	1.16	1.17	1.64
WPMAN	---	1.29	1.12	1.36
	1	1.29	1.12	1.39
	(1, t)’	1.30	1.15	1.54
C	---	1.28	1.18	1.27
	1	1.28	1.13	1.30
	(1, t)’	1.30	1.15	1.20
I	---	1.23	1.13	1.30
	1	1.23	1.13	1.28
	(1, t)’	1.24	1.13	1.30
N	---	1.15	0.88	1.04
	1	1.20	0.91	1.28
	(1, t)’	1.20	0.92	1.26

4. CONCLUDING COMMENTS

In this article we have analyzed the stochastic behaviour of eleven macroeconomic time series of the Swedish economy, (annually, 1861–1988), by means of using fractionally integrated techniques. These series were also examined by Englund et al. (1992) and Skalin and Te-

TABLE 5										
Testing the order of integration in the log transformed series with white nosie u_t										
Series	z_t	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
PY	---	24.08	24.84	20.62	10.21	2.68	**-0.98***	-2.79	-3.81	-4.46
	1	24.08	22.10	19.41	14.90	7.90	3.48	**1.02***	**-0.64***	-1.90
	(1, t)'	26.28	23.95	19.73	13.88	7.93	3.70	**1.02***	**-0.63***	-1.90
W	---	24.51	26.44	18.30	6.20	**0.53***	-2.14	-3.50	-4.28	-4.79
	1	24.51	22.27	19.31	15.92	6.86	2.04	**-0.03***	**-1.38***	-2.39
	(1, t)'	27.56	25.45	21.14	14.18	6.93	2.44	**0.08***	**-1.31***	-2.34
PMAN	---	23.89	23.60	14.93	6.50	**1.07***	-1.89	-3.43	-4.30	-3.83
	1	23.89	21.56	17.82	12.06	6.11	2.26	**-0.21***	-1.92	-3.11
	(1, t)'	24.18	21.26	16.75	11.23	6.12	2.36	**-0.19***	-1.91	-3.11
YMAN	---	25.24	12.31	8.11	3.43	**-0.43***	-2.64	-3.84	-4.54	-4.99
	1	25.24	22.28	17.05	6.44	**-0.01***	-2.05	-3.40	-4.26	-4.83
	(1, t)'	16.19	12.71	7.93	3.31	**-0.03***	-2.15	-3.45	-4.28	-4.83
Y	---	25.05	14.77	9.87	3.83	**-0.39***	-2.67	-3.87	-4.57	-5.01
	1	25.05	22.27	17.80	10.11	**0.07***	-1.90	-3.08	-3.89	-4.43
	(1, t)'	21.56	15.27	8.71	3.43	**0.04***	-1.96	-3.17	-3.94	-4.46
X	---	23.36	16.95	10.04	3.90	**-0.44***	-2.76	-3.95	-4.63	-5.01
	1	23.36	19.94	13.43	3.91	**-0.39***	-2.23	-3.35	-4.10	-4.60
	(1, t)'	22.49	16.83	9.00	2.95	**-0.40***	-2.24	-3.36	-4.09	-4.58
M	---	23.33	15.65	9.55	3.83	**-0.56***	-2.80	-3.96	-4.63	-5.06
	1	23.33	19.56	11.31	**0.99***	-1.95	-3.08	-3.75	-4.19	-4.51
	(1, t)'	20.22	13.01	5.16	**0.33***	-1.95	-3.09	-3.75	-4.18	-4.49
WPMAN	---	24.54	24.62	18.26	6.14	1.68	**-0.63***	-2.18	-3.25	-4.01
	1	24.54	21.77	16.81	8.22	2.46	**-0.07***	-1.77	-2.95	-3.77
	(1, t)'	21.36	16.42	10.89	6.08	2.47	**-0.05***	-1.78	-2.95	-3.76
C	---	25.07	14.48	9.68	3.82	**-0.41***	-2.69	-3.89	-4.59	-5.03
	1	25.07	22.26	14.36	6.73	**-0.61***	-2.15	-3.13	-3.80	-4.26
	(1, t)'	17.72	11.47	5.97	1.92	**-0.64***	-2.19	-3.16	-3.80	-4.26
I	---	25.11	13.81	9.00	3.88	**-0.11***	-2.38	-3.63	-4.37	-4.85
	1	25.11	21.77	14.06	**1.10***	-2.13	-3.32	-4.09	-4.60	-4.96
	(1, t)'	15.55	9.58	3.98	**0.11***	-2.08	-3.34	-4.10	-4.54	-4.86
N	---	25.49	13.97	9.59	3.72	**-0.48***	-2.71	-3.88	-4.56	-5.00
	1	25.49	22.26	17.19	10.71	4.24	**0.35***	-1.65	-2.77	-3.511
	(1, t)'	26.98	24.82	19.89	11.68	4.17	**0.15***	-1.70	-2.75	-3.48

* and in bold: Non-rejection values of the null hypothesis at the 95% significant level.

rasvirta (1999). In the first of these papers, they take first differences on the data to remove the nonstationarity while Skalin and Terasvirta (1999) proposed non-linear models. Here, we have used a version of the tests of Robinson (1994) that permits us to test unit and fractional roots with the possibility of including deterministic trends and with no effect on the standard normal

TABLE 6										
Testing the order of integration in the log-transformed data with Bloomfield (1) u_t										
Series	z_t	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
PY	---	14.09	12.31	9.02	3.84	**0.33***	**-1.62***	-2.68	-3.26	-3.62
	1	12.09	10.67	8.59	4.68	**0.42***	**-1.51***	-2.43	-2.99	-3.39
	(1, t)’	14.03	10.47	7.16	3.37	**0.35***	**-1.44***	-2.38	-2.98	-3.38
W	---	12.98	12.89	8.54	2.67	**-0.42***	-2.05	-2.88	-3.48	-3.79
	1	12.98	10.75	8.98	5.89	**0.05***	-2.16	-3.05	-3.40	-3.75
	(1, t)’	14.39	11.85	8.08	3.56	**-0.04***	-1.90	-2.90	-3.41	-3.65
PMAN	---	11.82	10.94	6.49	2.51	**0.04***	**-1.58***	-2.51	-3.18	-3.60
	1	11.82	9.90	7.07	2.86	**0.16***	**-1.37***	-2.10	-2.62	-3.01
	(1, t)’	11.41	8.42	5.30	2.16	**0.18***	**-1.25***	-2.07	-2.61	-3.00
YMAN	---	13.22	4.82	3.00	**1.19***	**-0.77***	-2.01	-2.88	-3.41	-3.77
	1	13.22	10.91	8.23	3.06	**-0.98***	-1.93	-2.59	-3.18	-3.54
	(1, t)’	6.19	4.28	2.14	**0.28***	**-1.17***	-2.02	-2.75	-3.23	-3.51
Y	---	13.00	5.93	4.10	**1.47***	**-0.54***	-1.92	-2.85	-3.39	-3.76
	1	13.00	10.91	8.51	4.51	-1.70	-2.59	-3.07	-3.53	-3.83
	(1, t)’	8.60	4.86	1.83	**-0.44***	-1.73	-2.60	-3.21	-3.64	-3.91
X	---	11.23	8.12	4.42	1.89	**-0.45***	-1.94	-2.29	-3.35	-3.79
	1	11.23	8.58	4.29	**0.24***	-1.80	-2.65	-3.13	-3.51	-3.85
	(1, t)’	9.74	6.44	2.44	**-0.31***	-1.82	-2.68	-3.16	-3.54	-3.82
M	---	11.04	7.21	4.09	**1.52***	**-0.51***	-1.95	-2.89	-3.45	-3.75
	1	11.04	8.00	3.43	**-1.42***	-3.09	-3.82	-4.24	-4.49	-4.75
	(1, t)’	7.38	3.56	**0.31***	-1.82	-3.11	-3.85	-4.19	-4.49	-4.63
WPMAN	---	12.15	11.68	6.65	**0.45***	**-1.48***	-2.38	-2.92	-3.31	-3.64
	1	12.15	10.42	7.56	1.77	**-1.20***	-2.21	-2.71	-3.17	-3.44
	(1, t)’	8.31	4.93	2.01	**-0.04***	**-1.29***	-2.19	-2.42	-3.16	-3.50
C	---	12.63	5.93	4.08	**1.43***	**-0.56***	-1.94	-2.75	-3.32	-3.70
	1	12.63	10.68	7.68	**1.22***	-2.66	-3.29	-3.64	-3.98	-4.22
	(1, t)’	5.75	2.24	**-0.10***	-1.76	-2.22	-3.35	-3.74	-4.02	-4.21
I	---	12.95	5.00	3.11	**1.00***	**-0.84***	-2.19	-2.91	-3.42	-3.79
	1	12.95	11.06	7.84	**0.59***	-2.09	-2.83	-3.27	-3.65	-3.99
	(1, t)’	5.24	2.88	**0.58***	**-1.05***	-2.11	-2.80	-3.29	-3.42	-3.44
N	---	13.39	5.58	4.05	**1.54***	**-0.69***	-2.04	-2.81	-3.35	-3.72
	1	13.32	10.62	7.42	2.49	**-0.31***	-3.16	-3.15	-3.65	-4.01
	(1, t)’	14.01	10.89	6.96	2.92	**-0.42***	-2.41	-3.23	-3.66	-3.90

* and in bold: Non-rejection values of the null hypothesis at the 95% significant level.

limit distribution of the tests. The results based on the original series suggest that the orders of integration are higher than 1, implying nonstationarity and non mean-reverting behaviour. This is important in the sense that the first differenced series still posses long memory (d > 0) and thus, the standard approach of taking first differences leads to series which are not I(0) imply-

TABLE 7										
Testing the order of integration in the log transformed series with Bloomfield (2) u_t										
Series	z_t	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
PY	---	6.19	5.73	4.04	3.10	**-0.05***	-1.93	-3.03	-3.85	-3.93
	1	8.19	7.99	4.51	**1.51***	**-0.88***	**-1.12***	-1.91	-3.14	-3.90
	(1, t)'	8.24	4.74	3.69	**0.77***	**-0.73***	-2.84	-3.71	-3.92	-3.97
W	---	10.44	10.04	6.99	3.20	**-1.07***	-1.96	-2.09	-2.59	-2.78
	1	9.44	5.19	4.93	3.36	**-0.17***	-2.39	-3.26	-3.77	-3.97
	(1, t)'	10.39	6.80	5.06	3.20	**0.24***	-1.65	-2.94	-3.57	-3.94
PMAN	---	7.27	5.83	2.55	2.00	**0.06***	-2.04	-2.62	-2.77	-4.14
	1	7.27	6.31	5.42	2.40	**-0.02***	-1.95	-2.61	-2.79	-3.34
	(1, t)'	7.84	6.31	2.88	**1.47***	**0.03***	-1.76	-2.55	-2.78	-3.33
YMAN	---	7.47	4.00	2.44	**1.01***	**-1.07***	-2.52	-3.31	-3.71	-3.91
	1	7.47	7.15	5.38	1.87	-2.89	-3.27	-3.63	-3.87	-3.97
	(1, t)'	1.75	1.57	**1.49***	**-0.81***	-2.30	-3.57	-3.89	-3.96	-4.01
Y	---	7.68	2.14	1.96	**1.63***	**-0.98***	-2.18	-3.41	-3.95	-4.19
	1	7.68	7.40	6.00	2.87	**-0.69***	-1.66	-1.89	-1.99	-4.04
	(1, t)'	4.10	3.21	**1.58***	**-0.33***	**-0.75***	-1.68	-2.12	-2.30	-2.33
X	---	5.93	4.19	1.76	**1.61*+**	**-1.11***	--1.69	-1.75	-2.28	-2.53
	1	6.93	6.63	2.96	1.66	-2.11	-2.88	-3.03	-3.47	-3.73
	(1, t)'	5.42	4.21	**1.05***	**-0.19***	-2.14	-2.92	-3.06	-3.39	-3.99
M	---	5.41	5.37	3.90	**1.11***	**-1.42***	-1.91	-2.08	-2.37	-2.61
	1	8.41	7.57	4.11	**1.42***	**-1.58***	-2.03	-2.12	-2.28	-2.75
	(1, t)'	5.64	3.92	**0.85***	**-0.46***	**-0.55***	-1.65	-2.12	-2.19	-2.79
WPMAN	---	8.77	7.99	6.11	**0.55***	-2.08	-2.74	-3.03	-3.14	-3.44
	1	8.77	6.79	6.02	2.94	**-1.45***	-2.18	-2.45	-2.54	-2.94
	(1, t)'	6.18	2.85	**0.38***	**0.19***	**-1.37***	-2.15	-2.49	-2.55	-2.91
C	---	6.95	2.18	1.95	**1.58***	**-1.02***	-1.70	-1.90	-1.96	-2.21
	1	7.96	6.71	5.63	3.57	-2.22	-3.41	-3.50	-3.55	-3.84
	(1, t)'	4.75	2.56	**1.19***	**-1.00***	**-1.39***	-2.05	-2.83	-3.10	-3.70
I	---	8.65	3.96	**1.28***	**0.55***	**-0.27***	-1.74	-2.51	-2.88	-3.05
	1	8.65	6.86	3.63	**0.02***	-2.06	-2.81	-3.15	-3.45	-3.69
	(1, t)'	1.98	**0.32***	**-0.14***	**-0.83***	-1.91	-2.97	-3.02	-3.14	-3.50
N	---	7.61	1.85	**1.52***	**1.45***	**-1.20***	-1.76	-2.01	-2.06	-2.29
	1	7.67	4.98	3.79	2.28	**-0.24***	**-0.68***	-2.28	-2.87	-2.98
	(1, t)'	9.21	7.41	5.18	3.41	**-0.41***	**-1.23***	-2.52	-2.60	-2.73

* and in bold: Non-rejection values of the null hypothesis at the 95% significant level.

ing biasedness in the estimation of the coefficients in regression models. However, looking at the log-transformed data, the unit root null hypothesis cannot be rejected in practically any series, suggesting that the growth rates are I(0) and do not possess long memory behaviour. It would be worthwhile proceeding to get point estimates of d, however, not only would this be

TABLE 8

Values of d which produces the lowest statistics in th log transformed series

Series	z_t	White noise	Bloomfield (1)	Bloomfield (2)
PY	---	1.16	1.04	0.99
	1	1.64	1.04	0.93
	(1, t)’	1.65	1.03	0.94
W	---	1.04	0.98	0.88
	1	1.49	1.00	0.89
	(1, t)’	1.50	1.00	1.04
PMAN	---	1.07	1.01	1.05
	1	1.47	1.01	1.00
	(1, t)’	1.47	1.02	1.01
YMAN	---	0.97	0.88	0.79
	1	1.00	0.88	0.80
	(1, t)’	1.00	0.77	0.62
Y	---	0.97	0.91	0.87
	1	1.00	0.89	0.90
	(1, t)’	1.00	0.70	0.71
X	---	0.97	0.95	0.84
	1	0.97	0.77	0.81
	(1, t)’	0.96	0.70	0.74
M	---	0.96	0.91	0.82
	1	0.80	0.66	0.80
	(1, t)’	0.78	0.53	0.72
WPMAN	---	1.17	0.80	0.79
	1	1.23	0.85	0.86
	(1, t)’	1.23	0.74	0.77
C	---	0.97	0.91	0.80
	1	0.95	0.81	0.75
	(1, t)’	0.93	0.47	0.60
I	---	0.99	0.87	0.79
	1	0.80	0.78	0.77
	(1, t)’	0.76	0.58	0.43
N	---	0.96	0.92	0.81
	1	1.28	0.95	0.81
	(1, t)’	1.27	0.97	0.82

computationally more expensive, but it is then in any case confidence intervals rather than point estimates which should be stressed, while available rules of inference seem to require preliminary integer differencing to achieve stationarity and invertibility. The approach used in this paper simply generates computed diagnostics for departures from any real value of d and thus, it is not surprising that when fractional hypotheses are entertained, some evidence supporting them may appears, because this might happen even when the unit root model is highly suitable. However, even though the practice of computing test statistics for a wide range of null hypotheses may lead to ambiguous conclusions, often the bulk of these hypotheses are rejected, suggesting that the optimal local power properties of the tests, shown by Robinson (1994) may be supported by reasonable performance against non-local alternatives. In that respect, a model selection criterion, perhaps based on diagnostic tests on the residuals should be established to determine which may be the most adequate specification for this and other macroeconomic time series.

This article can be extended in several directions. Other methods for estimating and testing the fractional differencing parameter d, like the semiparametric procedures of Geweke and Porter-Hudak (1982) or Robinson (1995a,b) may be employed. However, these methods may be too sensitive to the choice of the bandwidth parameter number, while Robinson's (1994) parametric procedure proposed here produces simple and clear results, with strong evidence in favour of the unit root models in case of the log-transformed series. In any case, the fact that the results we have obtained are robust to the model chosen for the disturbances, though, suggests that these semiparametric methods would produce very similar results to ours. Another possible extension is to examine these series in a multivariate context and this would lead to the study of cointegration. In case of a bivariate system, a neccessary condition for (fractional) cointegration is that both individual series must have the same order of integration but this is not a problem here in view of the strong evidence in favour of the unit root in the log-transformed series. Then, the tests of Robinson (1994) can be performed on the residuals from the cointegrating regression. A problem with this procedure is that the residuals are not actually observed but obtained from the cointegrating regression, and thus there might be a bias in favour of stationary residuals. Extensions of the multivariate version of the tests of Robinson (1994) which permit us to test fractional cointegration in a system-based model is also of interest. There exists a reduced-rank procedure suggested by Robinson and Yajima (2000), However, it is not directly applicable here, since that method assumes I(d) stationarity ($d < 0.5$) for the individual series while we consider series which are nonstationary. ■

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