KLAUS KULTTI

Price Formation by Bargaining and Posted Prices

ABSTRACT

I study markets with two types of agents. Sellers have an indivisible good for sale, and their reservation value is zero. Buyers are randomly matched with sellers, and they value the good at unity. Sellers may be matched with any number of buyers, and they may choose to determine the price of the good either by bargaining or by posting prices. These choices are relevant only when a seller meets exactly one buyer. If two or more buyers are matched to a seller the buyers engage in an auction. The agents may choose whether to go to markets with bargaining or posted prices. I show that both market structures are equilibria but that they do not co-exist. Markets with posted prices are shown to be the unique evolutionary stable equilibrium.

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1. INTRODUCTION

In complete frictionless markets prices and quantities traded are determined by supply and demand; equilibrium prices equate demand and supply. The story behind price formation is that of competition; if demand were greater than supply some agents could not get the goods they desire, and they would be willing to pay more for them which would increase price. This

KLAUS KULTTI, Professor
Helsinki School of Economics and Business Administration, Department of Economics
• e-mail: kultti@hkkk.fi

logic is rarely made explicit by describing how trades are consummated on the level of individual agents, and in a static setting it would be impossible to do so.

The model of complete markets completely abstracts from the trading mechanisms or institutions. In reality there seem to be three somewhat distinguishable trading mechanisms, namely auctions, bargaining, and posted prices. All of these have been used to study price formation in economywide markets. Rubinstein and Wolinsky (1985) study a model in which buyers and sellers are randomly matched in pairs, and they negotiate or bargain over the terms of trade. Posted prices are used extensively in the literature; search models starting from Diamond (1971) constitute a major example. Lu and McAfee (1996) study the relative performance of bargaining and auctions in a model where agents are randomly matched.

In a partial equilibrium setting the seller's choice between auctions and posted prices, and between bargaining and posted prices has been studied by Wang (1993, 1995). Even though the models are dynamic they are rather restrictive as the mechanisms are characterised by exogenously given costs, and they feature a monopolistic seller with exogenously given demand.

All the above models share the feature that sellers are assumed to be able to commit to a trading mechanism. This is not a satisfactory assumption when there are many agents. One would also expect the selling mechanism to affect the buyers' willingness to participate in the markets. To the extent that demand depends on the selling mechanism models with exogenously given demand are of limited interest.

My aim is to study both price formation and the performance of various trading mechanisms. It is clear that to study these matters one cannot adopt the frictionless market model. I use the random matching model of Lu and McAfee (1996) which has two attractive features. First, the meeting probabilities depend in a well specified manner on the numbers of buyers and sellers. Second, the agents do not necessarily meet pairwise but several, say, buyers may be matched to a single seller. These features are intimately connected, and they make possible meaningful modelling of different modes of trade. The agents are not given unjustified commitment power, or the selling mechanism is not postulated in such a way that it results in ineffciency except to the extent that comes from the meeting frictions.

First I briefly describe the model of Lu and McAfee (1996) since some of the assumptions in their article depict what I regard as shortcomings as to price determination. They study bargaining markets and auction markets. In both markets sellers are in fixed positions, and buyers are randomly distributed on them. In the bargaining markets sellers commit to determine the price of the object for sale by splitting the gains from trade which results in the same outcome as Rubinstein's (1982) alternating offers bargaining procedure where agents can leave the current partner in case of disagreement. If two or more buyers happen to be matched to the seller

Lu and McAfee assume that the bargaining partner is selected randomly amongst the buyers. This is dubious since the agreed price is such that anyone of the remaining buyers could offer the seller a higher price, and both of them would be better-off. One would expect that in the case of excess demand (more than one buyer) the buyers would compete for the good.

In auction markets sellers commit to sell the good in an auction. If a seller meets only one buyer then the buyer offers the seller his reservation value. Again, it is hard to see why the seller would commit to this kind of procedure when any alternative individually rational behaviour would make him at least as well-off. In particular, the seller could try to negotiate the price with the buyer.

To catch aspects of competition that characterise the perfectly competitive market model I use auctions. When two or more buyers meet a seller they are assumed to compete for the good for sale and engage in an auction. As the agents are randomly matched this means that when there are relatively few buyers auctions take place infrequently, and when there are relatively many buyers auctions happen all the time. In the latter case competition drives prices up. What happens in the former case depends on how price formation is modelled when a seller meets exactly one buyer. I assume that in this case the sellers are able to commit to a particular trading mechanism of which I study bargaining and posted prices. It is worth emphasising that the mechanisms are not pure; they are short for 'bargaining if one seller and one buyer, auction otherwise', and 'posted price if one seller and one buyer, auction otherwise'. Bargaining is modelled as a variant of the usual alternating offers game. To model posted prices in an interesting way I must say something about the way buyers choose the sellers they go to. This means that the agents are assumed to behave in a more sophisticated way than in the standard random matching model. Notice that the optimal selling mechanism is not studied but rather two well known selling mechanisms are compared.

It is also worth emphasising that evolutionary dynamics is used to select amongst equilibria for two reasons. First, it is a familiar and well understood method. Second, in the dynamic infinitely repeated setting where the set of players is somewhat vaguely defined it is also simple. There are no doubt other ways to do the same thing.

My scant empirical evidence suggests that the bargaining mechanism is used, for instance, in housing markets; many times the price is not announced at all, and depending on the number of willing buyers the seller engages in bargaining or the buyers outbid each other. When the prices must be announced (like in Finland) they are frequently so high that everybody understands that the final price is determined in bargaining or an auction type situation depending on the magnitude of demand. An example of posted price mechanism is an auction with a reservation price.

Related literature about the stability of trading mechanisms in a general equilibrium setting consists of Lu and McAfee (1996). They show that auction markets are the unique stable equilibrium, and dominate bargaining markets in this sense. Kultti (1998) shows that in the same setting posted price markets are equivalent to auction markets. Kultti (1997) studies price formation in a similar model where prices are determined by bargaining if exactly one buyer and one seller meet. Otherwise they are determined in an auction. Buyers and sellers are treated symmetrically in a sense that both may choose to search or wait for partners.

The rest of the article is organised as follows: In section 2 I present the model, and study the bargaining markets and posted price markets separately. In section 3 I determine the equilibria of the model, and study their stability. In section 4 I present conclusions.

2. THE MODEL

Consider markets with B buyers and S sellers where these numbers are large. Each seller has a unit of indivisible good for sale, and each buyer desires exactly one unit of this good. All sellers value the good at zero, and all buyers value the good at unity. These valuations can be regarded as reservation values in a static one period setting. In the dynamic setting the actual reservation values are determined endogenously.

I study two markets that may exist simultaneously. In both markets sellers are in fixed locations, and buyers are distributed on them randomly. In one market sellers post prices that are observed by buyers before they are matched with the sellers. If a seller meets exactly one buyer, and the buyer wants the good he has to pay the posted price. If a seller meets two or more buyers an auction ensues. I consider two types of auctions. In one auction buyers engage in a Bertrand-competition for the object, and the equilibrium price is such that all buyers are pushed to their reservation utility levels. In the other auction the buyers make offers to each other for the right to buy the object at the posted price so that the seller always gets just the posted price for the object.

In the other market sellers bargain on price if they meet exactly one buyer. Bargaining proceeds as in Rubinstein's (1982) alternating offers bargaining game with agents being able to leave each other. If a seller meets two or more buyers an auction ensues. Both buyers and sellers can decide which markets to enter. Agents that manage to trade exit the markets and are replaced by identical agents that on entrance decide which markets they go to. This guarantees the stationarity of the environment.

Time is discrete, and the agents have a common discount factor $\delta \in (0,1)$. The events within a period proceed in a fixed sequence: New sellers and buyers enter the markets, sellers post prices in the posted price market, buyers observe the prices, buyers are distributed on sellers in both markets, trading takes places, and those who trade exit the markets. Let us denote the ratio of buyers to sellers by $\theta = \frac{B}{S}$ which stays constant over time, the proportion of buyers in the posted price markets by x, and that of sellers by y. Then the proportion of buyers in the bargaining markets is $1-x$, and that of sellers $1-y$.

The number of buyers a seller meets is binomially distributed. Consider eg. posted price markets. There are xB buyers and yS sellers. As the buyers are, in equilibrium, distributed on the sellers independently with identical probabilities the probability that a fixed seller meets any particular buyer is $1/yS$. Thus the number of buyers a seller meets is distributed according to Bin(xB, $1/yS$). Analogously the number of buyers that a seller meets in an auction market is distributed according to Bin($(1-x)B$, $1/(1-y)S$). Since binomial distributions are awkward to deal with I approximate them with Poisson distributions. The approximation holds exactly in the limit when B and S approach infinity in such a way that their ratio remains constant. Since the number of buyers a seller meets is the crucial factor in our model the results do not change qualitatively even when the approximation is not perfect. In the posted price market I use a Poisson distribution with rate $\alpha = \frac{x}{y}\theta$, and in the bargaining market a Poisson distribution with rate $\beta \frac{1-x}{1-y}\theta$.

2.1. Posted price markets

In these markets the sellers post prices that buyers take as given (with the understanding that competition leads to an auction). This creates problems if nothing more is postulated since clearly the optimal pricing rule from the sellers' point of view is to post price equal to unity. Given that a fixed number of buyers are in the markets and they are randomly distributed on the sellers it does not pay to lower the price. This is a higly unsatisfactory way to think of posted price markets. One would like to introduce some elements of competition by letting the buyers choose which sellers they go to after they have received some information about prices, and by letting sellers observe each others' prices. This is not as straightforward as one might expect in this framework, and I post-pone the discussion to the end of this section. For the moment let us denote the price in the markets by p. I focus on situations in which every seller posts the same price.

If exactly one buyer appears he gets the good at the posted price. If two or more buyers appear the good is sold in auction. The seller meets no buyer with probability $e^{-\alpha}$, exactly one buyer with probability $\alpha e^{-\alpha}$, and two or more buyers with probability $1-e^{-\alpha}-\alpha e^{-\alpha}$. The buyer is the only buyer to meet the seller he is matched to with probability $e^{-\alpha}$ and with probability $1-e^{-\alpha}$ there are other buyers, too. The expected utilities of sellers and buyers, respectively, are

$$U_s^p = \delta\{e^{-\alpha}U_s^p + \alpha e^{-\alpha}p + (1-e^{-\alpha}-\alpha e^{-\alpha})(1 - U_b^p)\} \quad (1)$$

(2) $$U_b^p = \delta\{e^{-\alpha}(1-p) + (1-e^{-\alpha})\, U_b^p\}$$

From (1) and (2) one solves the expected utilities as a function of *p*.

(3) $$U_s^p = \frac{\delta e^{-\alpha}(\alpha - \delta\alpha + \delta - \delta e^{-\alpha})}{(1-\delta e^{-\alpha})(1-\delta + \delta e^{-\alpha})}\, p + \frac{\delta(1-\delta)(1-e^{-\alpha} - \alpha e^{-\alpha})}{(1-\delta e^{-\alpha})(1-\delta + \delta e^{-\alpha})}$$

(4) $$U_b^p = \frac{\delta e^{-\alpha}(1-p)}{1-\delta + \delta e^{-\alpha}}$$

Next I address the question about price determination. The basic idea is that there is competition in the markets, and consequently prices affect the number of buyers a seller meets. To this end I assume that buyers observe all prices and then decide independently which sellers they go to. If all sellers post the same price buyers are indifferent, and in equilibrium they choose a mixed strategy that puts equal weight to each seller. If buyers observe non-uniform prices they choose a mixed strategy that puts different weights to different sellers depending on the price they post. Given the distribution of prices the buyers choose the probabilities so that they constitute a Nash-equilibrium. In symmetric equilibrium there is price *p* such that no seller has an incentive to change his price if all others stick to *p*. The equilibrium price is determined considering one time deviations. This is sufficient since the buyers are matched afresh every period, they do not recognise previous partners, and they have no other ways of identifying the sellers but the prices they offer. Further, since exiting agents are replaced by identical agents the situation is similar in every period. Thus, any one seller can use a current period price decision to improve only his current period position.

Assume for a moment that there are B' buyers and S' sellers in the market so that $\alpha = \frac{B'}{S'}$ and that proportion *z* of the sellers deviates or is forced to deviate together. That more than one seller deviate simultaneously is just a modelling trick which makes analysis easier.

In equilibrium all sellers post price *p*. Consider proportion *z* of sellers who deviate for one period and post price p'. The buyers observe the prices and choose a mixed strategy $(\sigma, 1-\sigma)$ that determines whether they go to sellers with price p' or *p*. The mixed strategy is such that the buyers are indifferent between the sellers

(5) $$e^{-\alpha'}(1-p') + (1-e^{-\alpha'})\, U_b^p = e^{-\tilde{\alpha}}(1-p) + (1-e^{-\tilde{\alpha}})\, U_b^p$$

where $\alpha' = \frac{\sigma B'}{zS'}$ and $\tilde{\alpha} = \frac{(1-\sigma)B'}{(1-z)S'}$. In (5) the left hand side is the expected utility of a buyer who goes to a seller with price p′. If he manages to get the good at price p' he gets utility $1-p'$. If he ends up in auction he gets his expected utility given by (4). The right hand side is the utility of a buyer who goes to a seller with price p. Notice that the meeting probabilities change as a result of the deviation. Equation (5) determines the equilibrium value of the mixed strategy $(\sigma, 1-\sigma)$.

Deviators maximise $e^{-\alpha'} U_s^p + \alpha' e^{-\alpha'} p' + (1 - e^{-\alpha'} - \alpha' e^{-\alpha'})(1 - U_b^p)$. From (5) one can solve p' as a function of σ which yields the following objective function for the deviators

$$(6) \qquad e^{-\alpha'} U_s^p + 1 - e^{-\alpha'} - (1 - e^{-\alpha'}) U_b^p + \alpha' e^{-\tilde{\alpha}} U_b^p - \alpha' e^{-\tilde{\alpha}} (1 - p)$$

Instead of choosing p' one can think that deviating sellers maximise (6) by choosing σ. The first order condition for the maximum is

$$(7) \qquad -e^{-\alpha'} \frac{U_s^p}{z} + e^{-\alpha'} \frac{1}{z} - e^{-\alpha'} \frac{U_b^p}{z} + e^{-\tilde{\alpha}} \frac{U_b^p}{z} + \alpha' e^{-\tilde{\alpha}} \frac{U_b^p}{1-z} - e^{-\tilde{\alpha}} \frac{1-p}{z} - \alpha' e^{-\tilde{\alpha}} \frac{1-p}{1-z} = 0$$

In equilibrium the deviating sellers' maximising choice of price is p, which means that the deviators are in exactly the same situation as the non-deviators. This means that in equilibrium σ has to be such that $\alpha' = \tilde{\alpha} = \alpha$. Inserting this into (7) gives the equilibrium p as a function of z

$$(8) \qquad p = \frac{\delta(1 - e^{-\alpha} - \alpha e^{-\alpha}) - z(\delta - \delta e^{-\alpha} - \alpha)}{1 - \delta \alpha e^{-\alpha} - z + \alpha z}$$

Values of z close to zero can be interpreted as a competitive environment. The sellers have to price in such a way that not even a small number of sellers finds it profitable to deviate. Positive z means that deviation is possible only if many sellers do it simultaneously. In this case a deviating seller knows that he is adversely affected since other sellers deviate, too, and thus the sellers can sustain a higher equilibrium price as the costs of deviation are partly internalised. One can easily confirm that p is an increasing function of z. The standard test for a Nash-equilibrium is to consider one deviating agent. Letting z go to zero has similar spirit.

In the limit as z approaches zero the equilibrium price becomes

$$(9) \qquad p = \frac{\delta(1 - e^{-\alpha} - \alpha e^{-\alpha})}{1 - \delta \alpha e^{-\alpha}}$$

Notice that when there are very few buyers (alpha is close to zero) and demand is low, the price goes towards zero, and when there are many buyers (alpha grows without limit) the price tends to delta. This happens because sellers always have a risk of ending up with no partner. The price also behaves well in a sense that it is increasing in alpha.

Plugging (9) back into (3) and (4) gives the expected utilities of sellers and buyers

(10) $$U_s^p = \frac{\delta(1-e^{-\alpha}-\alpha e^{-\alpha})}{1-\delta\alpha e^{-\alpha}}$$

(11) $$U_b^p = \frac{\delta e^{-\alpha}}{1-\delta\alpha e^{-\alpha}}$$

Another possibility is that the seller gets in all cases the announced price but with multiple buyers the buyers auction off the right to buy the good at the announced price. If there are k buyers the buyer who gets to buy the object pays to the $k-1$ other buyers each $1/k$ of the available surplus. The sellers' and buyers' expected utilities are determined by the following equations

(12) $$V_s^p = \delta\{e^{-\alpha}V_s^p + (1-e^{-\alpha})p\}$$

(13) $$V_b^p = \delta\left\{e^{-\alpha}(1-p) + \alpha e^{-\alpha}\left[V_b^p + \frac{1}{2}(1-p-V_b^p)\right] + \frac{\alpha^2}{2!}\left[V_b^p + \frac{1}{3}(1-p-V_b^p)\right] + \ldots\right\}$$

The interpretation of the equations is analogous to the previous case. The available surplus is $1-p-V_b^p$, and the buyers get an equal share of it regardless of who gets the object; with e.g. k buyers anybody who offers the other buyers anything less than one kth of the surplus earns himself more than one kth of the surplus. Consequently, it pays to increase the price in order to win the object at price p. Anybody who offers the other buyers more than one kth of the surplus earns himself less than one kth of the surplus. Thus the equilibrium price in the auction is to offer exactly one kth of the surplus to the others. It is easy to solve the expected utilities explicitly

(14) $$V_s^p = \frac{\delta(1-e^{-\alpha})}{1-\delta e^{-\alpha}}\,p$$

(15) $$V_b^p = \frac{\delta(1-e^{-\alpha})}{\alpha(1-\delta)+\delta(1-e^{-\alpha})}\,(1-p)$$

The equilibrium price is determined exactly as before, and it turns out

(16) $$p = \frac{(1-\delta e^{-\alpha})(1-e^{-\alpha}-\alpha e^{-\alpha})}{(1-e^{-\alpha})(1-\delta\alpha e^{-\alpha})}$$

Plugging this price into (14) and (15) yields exactly the same result as the previous case where the object (instead of the the purchasing right) is auctioned. The price, however, is different; when the seller always gets exactly the posted price the buyers know that in case there are many of them all of them get their share of the surplus immediately. The seller should be able to charge a higher price, and comparing (9) and (16) one can see that this is the case, indeed.

2.2. Bargaining markets

In the bargaining markets sellers negotiate the price with a buyer when exactly one buyer appears. When two or more buyers appear an auction is held like in the posted price market. Negotiations about price are modelled as an alternating offers bargaining game (Rubinstein, 1982). With equal probabilities either of the agents is selected to make a proposal, and if the other agent accepts it trade is consummated. If the other agent does not accept the offer time proceeds to the next period. Then the same procedure is repeated but it is the other agent who makes the proposal. However, both the buyer and the seller can leave the current partner. In equilibrium the buyer does leave if it is the seller's turn to make the offer, and the seller ends the relationship if it is the buyer's turn to make the offer. Leaving or staying with the current partner does not affect the buyer's chances to get a good either in negotiations or in an auction, but staying with seller who makes an offer is like ending up in auction. Thus, in equilibrium the bargaining procedure proceeds as if in each period the proposer were selected by the flip of a coin. This results in the equal division of the surplus in expected terms.

Let us denote the buyers' proposal by $(v,1-v)$ and the sellers' proposal by $(w,1-w)$ where the first co-ordinate indicates the buyers' share of the surplus. I focus on so called semi-stationary strategies (Rubinstein and Wolinsky, 1985) where all agents use the same strategy against all opponents. In subgame perfect equilibrium the proposer makes an offer that leaves the respondent indifferent between accepting and rejecting. This means that the respondent is offered his reservation utility which is the same as his expected utility before being matched with anybody. Formally

$$(17) \qquad w = U_b^n$$

$$(18) \qquad 1 - v = U_s^n$$

where the superindex n signifies negotiations. Denote the proportion of buyers to sellers in this market by β. The sellers' and buyers' expected utilities are determined by the following equations

$$(19) \qquad U_s^n = \delta\left\{e^{-\beta}U_s^n + \beta e^{-\beta}\left[\frac{1}{2}(1-v) + \frac{1}{2}(1-w)\right] + (1 - e^{-\beta} - \beta e^{-\beta})(1 - U_b^n)\right\}$$

$$(20) \qquad U_b^n = \delta\left\{e^{-\beta}\left(\frac{1}{2}v + \frac{1}{2}w\right) + (1 - e^{-\beta})\,U_b^n\right\}$$

From (17)–(120) one solves for the values of interest

(21) $$U_s^n = \frac{\delta(2-2e^{-\beta}-\beta e^{-\beta})}{2-\delta e^{-\beta}-\delta\beta e^{-\beta}}$$

(22) $$U_b^n = \frac{\delta e^{-\beta}}{2-\delta e^{-\beta}-\delta\beta e^{-\beta}}$$

3. EQUILIBRIA

For either market structure the agents' optimal behaviour is known. In the beginning of a period the agents decide which markets they go to. In equilibrium no agent should be able to do better by behaving differently, i.e. by going to the other market. There exist three possible equilibria in the economy: i) Only posted price markets exist, ii) only bargaining markets exist, iii) posted price and bargaining markets exist simultaneously. It is clear that either market by itself, i.e. case i) or ii) is an equilibrium since no agent can deviate profitably by going to the other inactive market. To determine whether both markets exist simultaneously in equilibrium I study sellers' equilibrium curve (SE) and buyers' equilibrium curve (BE) that are got by equating (10) and (21) as well as (11) and (22)

(23) $$\frac{e^{\alpha}-1-\alpha}{e^{\alpha}-\delta\alpha} = \frac{2e^{\beta}-2-\beta}{2e^{\beta}-\delta-\delta\beta}$$

(24) $$\frac{\delta}{e^{\alpha}-\delta\alpha} = \frac{\delta}{2e^{\beta}-\delta-\delta\beta}$$

Dividing the LHS and RHS of (23) by the corresponding sides of (24) one gets

(25) $$e^{\alpha}-\alpha = 2e^{\beta}-1-\beta$$

and then using (24) one can solve for alpha

(26) $$\alpha = 1+\beta$$

Plugging (26) back to (25) and manipulating a bit one finds that in equilibrium the following equation has to hold

(27) $$e = 2$$

but this is clearly untrue. From this I conclude that SE and BE never intersect, and thus posted price markets and bargaining markets do not co-exist in equilibrium.

Proposition 1. Either market by itself constitutes an equilibrium. There is no equilibrium in which the two markets exist simultaneously.

This is an interesting and not obvious result. When competitive forces are introduced by the auction trick only either posted prices or bargaining is used in equilibrium. This is in contrast to related models (Lu and McAfee, 1996; Kultti 1997b) where sellers commit to a particular trading mechanism. In these models there are at least some parameter values for which two markets co-exist in equilibrium. I should like to say something about which of the two markets is a more likely configuration. One important aspect is the performance of the trading institution from the participants point of view which boils down to the division of surplus. To gain insight into these matters I study the situation from an evolutionary perspective. The analysis is similar to that in Lu and McAfee (1996), and is based on the replicator dynamics of evolutionary theory (Nachbar, 1990).

The entrants to the economy decide which market they go to based on how well other agents of their type did in the previous period. This leads to an increase in the relative share of buyers (sellers) in markets where buyers (sellers) did better. Since the model is discrete the adjustment process is discrete, too, and this may lead to cycles. I ignore this complication, and regard the dynamics as a continuous process. This can be achieved in various ways. Perhaps it is simplest to think that only a small fraction of the total population is active in any period. One can also postulate that of the new entrants only a fraction is free to choose a behaviour different from that of the exiting agents.

Before proceeding I want to emphasise that evolutionary dynamics is only one way to study stability of equilibria or to conduct equilibrium selection. It is based on non-optimising myopic behaviour, but it is well-understood, and provides a useful way of studying the relative performance of competing institutions. Myopicity here has two consequences. First, the entrants go to the market where their type fared best in the previous period. Thus, they only look at the latest pay-off and do not take into account the future pay-offs that their entry decisions influence. Second, the pay-offs of the agents in the markets are determined by calculating their stationary life time utilities. Myopic agents are assumed to think that the situation remains the same forever even outside the stationary state. This assumption, of course, does not turn out correct. The agents understand that the economy continues for ever but not much more; they could be called myopically farsighted.

The analysis proceeds in the following way: First I determine the relative positions of SE and BE in an x-y-plane. Then I study disequilibrium states, i.e. points off SE and BE. I deter-

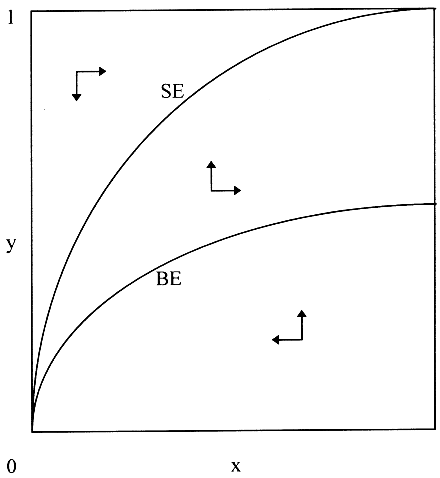

FIGURE 1. $\theta < \theta_0$.

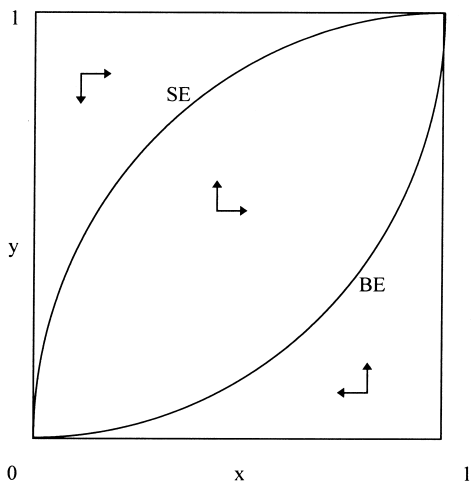

FIGURE 2. $\theta \geq \theta_0$.

mine in which markets the agents do better and from this I can conclude where new entrants go. This determines whether x and y increase or decrease. In two dimensions the analysis is easily conducted graphically. The following lemmata give the results needed. Even though the proportions of agents in various markets change in time as a result of the adjustment the time dependence is not shown explicitly. All the proofs are relegated to the appendix. The easiest way to see the content of the following results is to consult Figures 1 and 2 where the position of the SE and BE curves is shown. The arrows indicate the direction of adjustement. Notice that only the relative position of the curves is important, and the figures are not accurate in other respects.

Lemma 1. SE and BE are increasing, $\frac{dy}{dx}\big|_{SE} > 0$ and $\frac{dy}{dx}\big|_{BE} > 0$.

Consider function $f(\theta) = e^{\theta} - \delta\theta - 2 + \delta, \theta \geq 0$. It has a unique zero θ_0 which, of course, depends on δ.

Lemma 2. SE and BE always contain point (0,0), and SE always contains point (1,1). If $\theta < \theta_0$ BE contains point $(1,y)$ for some $y < 1$. If $\theta \geq \theta_0$ BE contains point (1,1).

Lemma 3. SE is always above BE.

Lemma 4. Above BE buyers prefer posted price markets, i.e. in the area above BE $U_b^p > U_b^n$, and x increases. Below BE buyers prefer bargaining markets, i.e. in the area below BE $U_b^p < U_b^n$, and x decreases. Above SE sellers prefer bargaining markets, i.e. in the area above SE $U_s^n > U_s^p$, and *y* decreases. Below SE sellers prefer posted price markets, i.e. in the area below SE $U_s^p > U_s^n$, and y increases.

Proposition 2. Posted price markets is the unique stable equilibrium.

The message of proposition 2 is very clear; to the extent that evolutionary dynamics is considered sensible markets with posted prices is a better institution than markets where agents bargain over the prices. Of course, the result makes sense only if one is willing to accept the modelling of competitive forces by auction. The determination of posted prices is based on the fact that buyers observe the prices before they decide which sellers they go to, and that sellers know this. When the frictions of the market are such that buyers do not know prices, but have to search for the right price or good the result is not applicable. It is worth emphasising that the result does not follow from the fact that the choice set of the sellers in the posted price markets can be thought larger than in the bargaining markets. In principle the sellers could offer the price that corresponds to the outcome of the bargaining. However, individual sellers would like to deviate from this. Since their choice set is large in the sense that they can choose other prices, too, they actually deviate. If all but one seller had to choose the price that corresponds to the bargaining outcome then this seller could, of course, improve his situation, but this does not hold good if the choice set of all other sellers becomes larger, too.

One motivation for the price formation stories examined is that I do not want to make sellers able to commit to a price mechanism when competition should drive price up, or when the commitment is disadvantageous to the seller. Selling goods only in auctions falls in the latter category when the seller meets only one buyer. However, it is easy enough to calculate the expected utilities of buyers and sellers also for this case. Let us denote the ratio of buyers to sellers by γ. Then the utilities are determined by

$$(28) \qquad U_s^a = \delta\,[e^{-\gamma} + \gamma e^{-\gamma}) U_s^a + (1 - e^{-\gamma} - \gamma e^{-\gamma})(1 - U_b^a)]$$

$$(29) \qquad U_b^a = \delta\,[(e^{-\gamma}(1 - U_s^a) + (1 - e^{-\gamma})\, U_b^a]$$

From (28) and (29) one can solve the expected utilities

$$(30) \qquad U_s^a = \frac{\delta(1 - e^{-\gamma} - \gamma e^{-\gamma})}{1 - \delta\gamma e^{-\gamma}}$$

$$(31) \qquad U_b^a = \frac{\delta\, e^{-\gamma}}{1 - \delta\gamma e^{-\gamma}}$$

Comparing (30) and (31) to (10) and (11) one notices that they are of exactly same form.

This means that posted price markets are equivalent to markets where all trades are consummated in auctions. A similar result is obtained in Kultti (1998) in a setting where sellers are assumed to be able to commit to a trading mechanism. Pure auction markets and posted price markets are equivalent since in both markets the surplus is divided in the same way. The logic behind this division, however, differs; in posted price markets the sellers' pricing affects the demand they face while in pure auction markets buyers are just randomly distributed on the sellers. The two markets are equivalent only in equilibrium.

4. CONCLUSION

I study price formation in a random matching model, and I concentrate on two common mechanisms of price determination. One is bargaining and the other posted prices. The focus is on markets with many participants, and thus they are not endowed with commitment powers that are not reasonable in competitive environments. Competition is modelled via auctions; when two or more buyers desire a good they compete for it in a Bertrand-competition like auction. When a seller meets exactly one buyer he may commit to bargain over the price, or to charge a posted price. Bargaining markets and posted price markets are both equilibria but they do not co-exist. When agents are allowed to choose between markets the posted price markets turn out to be the unique evolutionary stable equilibrium.

The result hints to the superiority of posted prices over bargaining. This result is in accordance with a result by Kultti (1998) where posted prices and auctions are shown to be equivalent, and a result by Lu and McAfee (1996) where auctions are shown to dominate bargaining. Indeed, the posted price markets of this article are equivalent to markets where only auctions are held. The results suggest that bargaining should not survive as an equilibrium institution. Casual evidence shows to the contrary, as bargaining appears to be a common method of price determination in many markets; used cars and housing markets are two prominent examples. These markets are characterised by agents who have different valuations and asymmetric information about the objects for sale. The main obstacle for the study of these situations seems to be the modelling of bargaining under asymmetric information as this can be done in numerous ways, and the models typically possess a large number of equilibria.

It should also be noted that there are many ways to model posted prices. One could, for instance, think that buyers may move from posted price markets to the bargaining markets after having observed prices which they do not observe before entering. This would probably make the bargaining markets more attractive to buyers if they observe only the price of the seller they are matched with; since sellers would not compete against each other the sellers would just charge a price that makes buyers indifferent between changing markets. In case there are moving costs this price is such that the buyers would not enter posted price markets in the first place. If the buyers could observe all the prices in the posted price market the sellers would charge a mark-up that equals the moving costs. The more sophisticated the interactions between buyers, sellers, prices and markets the more structure must be introduced to the model. I think that a particularly interesting feature of the way I model posted price mechanism is that it turns out to be equivalent to pure auction markets.

A further point to be noted is that I have focused only on stationary states; however one could also study a non-stationary economy where the population shares of the agents change from one period to the other. Then it is not immediate that there exists only one market in equilibrium. This would be an interesting situation to study but it seems much more complicated than the present analysis. Obviously the definition of equilibrium requires dynamic self-fulfilling expectations and the agents' optimising behaviour with respect to these expectations. Further issues would also arise; the prices in one period could serve as signals about the future pricing behaviour, and pricing would also affect the number of entrants instead of only the distribution of present buyers on the present sellers. Because of these complications the non-stationary case would constitute a separate study of its own. ■

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Appendix

Recall that $\alpha = \frac{x}{y}\theta$ and $\beta = \frac{1-x}{1-y}\theta$

Proof of Lemma 1.

Totally differentiating both sides of (23) and solving for the derivative gives

$$\text{(A1)} \qquad \frac{dy}{dx}\Big|_{SE} = \frac{\frac{1}{y}(A-B) + \frac{1}{1-y}(C-D+E)}{\frac{x}{y^2}(A-B) + \frac{1-x}{(1-y)^2}(C-D+E)}$$

where $A = e^{\alpha}(2+\beta-\delta-\delta\beta)$, $B = 2e^{\beta} - 2\delta e^{\beta} + \delta$, $C = 2e^{\beta} - \delta$, $D = (1-\delta)e^{\alpha}$, and $E = 2(1-\delta)\alpha e^{\beta}$. Using (23) one can easily confirm that both $A-B$ and $C-D+E$ are positive. Simplifying and totally differentiating both sides of (24) and solving for the derivative gives

$$\frac{dy}{dx}\Big|_{BE} = \frac{\frac{1}{y}(e^{\alpha}-\delta) + \frac{1}{1-y}(2e^{\beta}-\delta)}{\frac{x}{y^2}(e^{\alpha}-\delta) + \frac{1-x}{(1-y)^2}(2e^{\beta}-\delta)}$$

which is clearly positive. ■

Proof of Lemma 2.

I examine the behaviour of BE when one of the co-ordinates approaches its end points. The idea is to determine whether it is possible for the curves to exit the unit square via other points than *(0,0)* and *(1,1)*. This is done by letting one of the co-ordinates approach either zero or unity in turns.

i) Let $(x,y) \to (0,y)$. (24) becomes $1 = 2^{\frac{\theta}{1-y}} - \delta - \delta\frac{\theta}{1-y}$ which cannot hold as the RHS is always greater than unity.

ii) Let $(x,y) \to (1,y)$. (24) becomes $e^{\frac{\theta}{y}} - \delta\frac{\theta}{y} = 2-\delta$. This is of the form $f(\omega) = e^{\omega} - \delta\omega - 2 + \delta = 0$ where $\omega = \frac{\theta}{y}$. Funtion *f(ω)* has a unique zero $\omega_0 \in (0,1)$, and thus BE goes to $(1,y)$ where $y<1$ as long as $\theta < \omega_0$. Notice that ω_0 depends on δ and is increasing in δ.

iii) Let $(x,y) \to (x,1)$. (24) becomes $e^{x\theta} - \delta x\theta = 2e^{\infty} - \delta - \delta\infty$ which cannot hold.

iv) Let $(x,y) \to (x,0)$. (24) becomes $e^{\infty} - \delta\infty = 2e^{(1-x)\theta} - \delta - \delta(1-x)\theta$ which cannot hold.

The analysis of the behaviour of SE as one of the co-ordinates approaches its end points follows analogous lines.

i) Let $(x,y) \to (0,y)$. (23) becomes $0 = \dfrac{2e^{\frac{\theta}{1-y}} - 2 - \dfrac{\theta}{1-y}}{2e^{\frac{\theta}{1-y}} - \delta - \delta\dfrac{\theta}{1-y}}$ which cannot hold.

ii) Let $(x,y) \to (1,y)$. (23) becomes $0 = \dfrac{e^{\frac{\theta}{y}} - 1 - \dfrac{\theta}{y}}{e^{\frac{\theta}{y}} - \delta\dfrac{\theta}{y}}$ which cannot hold.

iii) Let $(x,y) \to (x,1)$. (23) becomes $\dfrac{e^{x\theta} - 1 - x\theta}{e^{x\theta} - \delta x\theta} = 1$ which cannot hold as the LHS is always less than unity.

iv) Let $(x,y) \to (x,0)$. (23) becomes $1 = \dfrac{2e^{(1-x)\theta} - 2 - (1-x)\theta}{2e^{(1-x)\theta} - \delta - \delta(1-x)\theta}$ which cannot hold as the RHS is always less than unity. ■

Proof of Lemma 3.

Since BE and SE are continuous in θ and they do not intersect one of them has to be above the other for all values of θ. According to Lemma BE contains point $(1,y)$, $y<1$, for small values of θ, and thus the only possibility is that SE is above BE. ■

Proof of Lemma 4.

Since BE and SE are continuous, as well as, U_b^p, U_b^n, U_s^p and U_s^n it is sufficient to study two points. I evaluate the expected utilities at points (0,1) and (1,0). The former is above BE and SE, and the latter below BE and SE. At (0,1) $U_b^p = \delta > U_b^n = 0$, and $U_s^p = 0 < U_s^n = \delta$. At (1,0) $U_b^p = 0 < U_b^n = \dfrac{\delta}{2-\delta}$, and $U_s^p = \delta > U_s^n = 0$. ■

Proof of Proposition 2.

Figures 1 and 2 depict the two relevant cases. The arrows indicate the direction of adjustment. Notice that only the relative position of the SE and BE curves is important, and the pictures are not accurate as to their actual shape. ■